



HAL
open science

The Hybrid Recharge Delayed Oscillator: A More Realistic El Niño Conceptual Model

Takeshi Izumo, Maxime Colin, Fei-Fei Jin, Bastien Pagli

► **To cite this version:**

Takeshi Izumo, Maxime Colin, Fei-Fei Jin, Bastien Pagli. The Hybrid Recharge Delayed Oscillator: A More Realistic El Niño Conceptual Model. *Journal of Climate*, 2024, 37 (9), pp.2765-2787. 10.1175/JCLI-D-23-0127.1 . ird-04577171

HAL Id: ird-04577171

<https://ird.hal.science/ird-04577171>

Submitted on 16 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

20 The El Niño Southern Oscillation (ENSO) is the leading mode of climate interannual
21 variability, with large socioeconomical and environmental impacts, potentially increasing with
22 climate change. Improving its understanding may shed further light about its
23 predictability. Here we revisit the two main conceptual models for explaining ENSO cyclic
24 nature, namely, the Recharge Oscillator (RO) and the advective-reflective Delayed Oscillator
25 (DO). Some previous studies have argued that these two models capture similar physical
26 processes. Yet, we show here that they actually capture two distinct roles of ocean wave
27 dynamics in ENSO's temperature tendency equation, using observations, reanalyses, and
28 Climate Model Intercomparison Project (CMIP) models. The slow recharge/discharge process
29 mostly influences central-eastern Pacific by favoring warmer equatorial undercurrent and
30 equatorial upwelling, while the 6-month delayed advective-reflective feedback process
31 dominates in the western-central Pacific. We thus propose a hybrid Recharge Delayed
32 Oscillator (RDO) that combines these two distinct processes into one conceptual model, more
33 realistic than the RO or DO alone. The RDO eigenvalues (frequency and growth rate) are
34 highly sensitive to the relative strengths of the recharge/discharge and delayed negative
35 feedbacks, which have distinct dependencies to mean state. Combining these two feedbacks
36 explains most of ENSO frequency diversity among models. Thanks to the two different spatial
37 patterns involved, the RDO can even capture ENSO spatiotemporal diversity and complexity.
38 We also develop a fully nonlinear and seasonal RDO, even more robust and realistic,
39 investigating each nonlinear term. The great RDO sensitivity may explain the observed and
40 simulated richness in ENSO's characteristics and predictability.

SIGNIFICANCE STATEMENT

42 El Niño and La Niña events, and related southern oscillation, cause the largest year-to-year
43 variations of earth's climate. Yet the theories behind them are still debated, with two main
44 conceptual models, the recharge oscillator and the delayed oscillator. Our purpose is here to
45 address this debate by developing a more realistic theory, a hybrid Recharge Delayed
46 Oscillator. We show how simple yet realistic it is, with equivalent contributions from the slow
47 recharge process and from the faster delayed feedback. It even captures the observed El Niño
48 and La Niña diversity in space and in frequency. Future studies could use the simple theoretical
49 framework provided here to investigate the El Niño Southern Oscillation in observations,
50 theories, climate models diagnostics and forecasts, and global warming projections.

51 **1. Introduction**

52

53 The El Niño Southern Oscillation (ENSO) is the leading mode of climate interannual
54 variability, with large socioeconomical and environmental impacts, and with its extremes
55 possibly increasing with global warming (e.g. Cai et al. 2021). ENSO basic amplification
56 dynamics has been fairly well understood (e.g. Neelin 1998, Wang and Picaut 2004, Clarke
57 2008 reviews). Yet there are still debates on the main mechanisms at play for ENSO phase
58 reversal and related quasi-cyclic nature, on ENSO theories / conceptual models (e.g. Graham
59 et al. 2015; Santoso et al. 2017; Timmermann et al. 2019; Jin et al. 2020), and on how to
60 implement ENSO diversity in these conceptual models (Fang and Mu 2018, Geng et al. 2020,
61 Chen et al. 2022, Thual and Dewitte 2023). ENSO diversity, a continuum from extreme Eastern
62 Pacific (EP) El Niño events to western-Central Pacific (CP) El Niño Modoki (e.g. Trenberth
63 and Stepaniak 2001, Ashok et al 2007, Takahashi et al. 2011, Capotondi et al. 2015, 2020), and
64 even encompassing La Niña events (Monselesan et al., submitted), is crucial to
65 understand/forecast ENSO global impacts (e.g. Johnson and Kosaka 2016), e.g. on tropical
66 cyclones activity in the Pacific, notably over vast French Polynesia (Vincent et al. 2011; Pagli
67 et al., in prep.).

68 ENSO events usually initiate in boreal spring, develop in summer and autumn and then
69 decay in the next spring (e.g. Bunge and Clarke 2009, Fang and Zheng 2018). They develop
70 because of the Bjerknes positive feedback. Positive Sea Surface Temperature (SST) anomalies
71 in the central-eastern equatorial Pacific (T_E) enhance deep atmospheric convection in the
72 central Pacific, resulting in westerly wind stress anomalies in the western-central Pacific (τ_x ;
73 Clarke 1994). These in turn force equatorial downwelling Kelvin waves with eastward surface
74 currents that advect the warm pool eastern edge eastward (e.g. Picaut et al. 1996; Vialard et al.
75 1998) and thermocline depth (h) positive anomalies towards the central-eastern equatorial
76 Pacific, thereby reinforcing the initial SST anomalies. This positive feedback mechanism is
77 offset by several negative instantaneous and delayed feedbacks. The delayed negative
78 feedbacks result from oceanic dynamics: 1) a slow negative feedback related to the wind-driven
79 slower equatorial basin adjustment to El Niño westerlies that acts to create a discharge (and a
80 recharge in the La Niña easterlies case) of Warm Water Volume (WWV)/Oceanic Heat Content
81 (OHC, proportional to h ; e.g. Jin 1997a,b; Meinen and McPhaden 2000; Clarke 2010; Zhu et
82 al. 2017; Izumo et al. 2019; Izumo and Colin 2022) of the southwestern and equatorial Pacific
83 (Fig. 1b), 2) a rapid ~6-month delayed advective-reflective feedback through equatorial

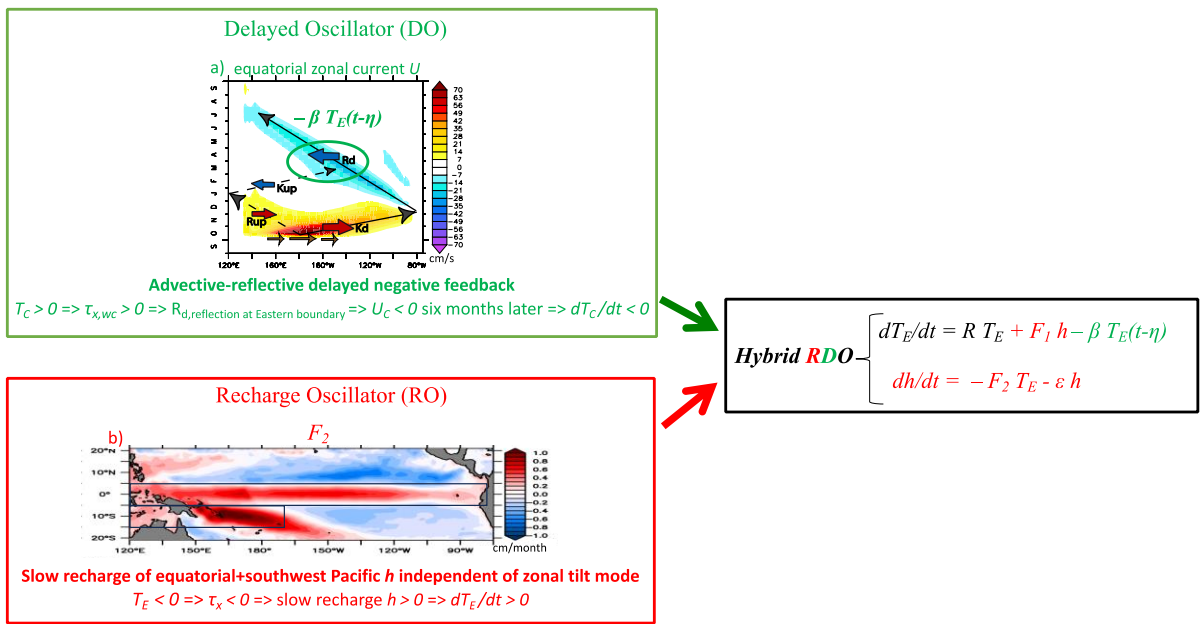
84 oceanic wave reflections (e.g. Boulanger and Menkes 2001; Boulanger et al. 2004; cf. Fig. 1a
85 and section 3).

86 Two main groups of theories have emerged to explain the quasi-cyclic nature of ENSO,
87 based on the aforementioned negative oceanic feedbacks: 1) the Recharge Oscillator (RO; e.g.
88 Wyrтки 1985; Jin 1997a,b; Meinen and McPhaden 2000; Clarke et al. 2007, 2010; Thual et al.
89 2013), 2) the delayed oscillator (Suarez and Schopf 1988, Battisti and Hirst 1989) and its
90 improved version emphasizing the crucial role of Indo-Pacific warmpool eastern edge, the
91 advective-reflective Delayed Oscillator (DO; Picaut et al. 1997). An in-depth description of the
92 RO and DO is given in section 3. Other conceptual models, e.g. the unified oscillator (Wang
93 2001), have been shown to be less realistic (Graham et al. 2015).

94 Some studies question the RO and suggest that the DO is more realistic (e.g. Linz et al.
95 2014, Graham et al. 2015). Yet, the recharge/discharge process is also part of ENSO cycle,
96 both in observations and models. One reason for this “RO vs DO” debate is that RO and DO
97 have been seen as two different formal ways to represent the same wave adjustment process
98 favoring an ENSO phase reversal (e.g. Jin 1997ab; Jin and An 1999). But actually, they mainly
99 involve different physical processes with different timescales (Fig. 1). For the RO, it is the slow
100 (~1-2 years) equatorial basin adjustment (e.g. Izumo et al. 2019, their Suppl. Fig. S4)
101 influencing central-eastern Pacific SST through the thermocline feedback (e.g. Wyrтки 1985,
102 Jin 1997ab), and through heat transport changes (Izumo 2005; Ballester et al. 2015, 2016). For
103 the DO, it is the faster (~6-month) equatorial wave reflection influencing central Pacific SST
104 through the zonal advective feedback (Vialard et al. 1998). So, is it really fair to consider the
105 RO and DO as a single mechanism while they capture physical distinct processes and have
106 different impacts on the T_E tendency equation?

107 Here we show, by analyzing observations/reanalyses (section 4b) and 79 CMIP models
108 (Climate Model Intercomparison Project; section 4c), that we should keep both the recharge
109 and advective-reflective delayed feedbacks as two distinct crucial processes in the ENSO
110 temperature tendency equation, without considering them as formally identical. By keeping
111 both of them, we obtain a more physically-based and more realistic ENSO conceptual model,
112 a hybrid Recharge Delayed Oscillator (RDO; schematics in Fig. 1; derivation of its equations
113 in section 4a). We then analyze the behavior of this RDO. Section 4d explores the eigenvalues
114 dependency to parameters, within the observations and CMIP parameter space. Section 4e first
115 investigates the RDO response to stochastic forcing and related spectral response. It then shows
116 how ENSO characteristics (amplitude and spectrum) in the RDO framework are highly
117 sensitive to the relative strengths of the recharge and delayed negative feedback. Section 4f

118 shows that the RDO can, even its simplest form, capture some ENSO spatiotemporal diversity,
 119 if we take into account the spatial dependency of these feedbacks. Section 5 then improves the
 120 simplest RDO framework, by adding: 1) the seasonal cycles of RDO parameters (section 5a),
 121 2) asymmetries/non-linearities, e.g. quadratic terms and a multiplicative noise (section 5b).
 122 Section 5 confirms the robustness of the RDO framework, with the non-linear seasonal RDO
 123 version being even more realistic than the linear RDO version. Finally, section 6 summarizes
 124 the results and discusses mechanisms, possible improvements of the RDO, implications and
 125 perspectives.



126
 127 **Fig. 1. Schematics of the Recharge Delayed Oscillator (RDO) principle, combining the advective-**
 128 **reflective Delayed Oscillator (DO; upper green; representing mainly the zonal feedback; Picaut et al.**
 129 **1997) and the Recharge Oscillator (RO; bottom red; representing the slow recharge process; Jin**
 130 **1997ab). a)** equatorial zonal current response (in the upper oceanic layer, color shading and red/blue arrows)
 131 to a westerly pulse in early October in the western-central Pacific (brown arrows), with equatorial waves
 132 (black arrows for downwelling waves, dashed arrows for upwelling waves) and their reflections indicated
 133 schematically (in particular the downwelling Kelvin wave K_d with positive zonal current reflecting at the
 134 eastern boundary as a downwelling Rossby wave R_d with opposite negative current; see Izumo et al. 2016
 135 for details on a similar idealized pulse experiment starting in January). **b)** slow recharge of h_{ind} (SLA
 136 independent of the fast zonal tilt mode) forced by La Niña easterly anomalies (i.e. the $-F_2 T_E$ term obtained
 137 by multivariate regression in obs1; vice versa for a discharge during El Niño). T_C and U_C denote central
 138 equatorial Pacific SST and zonal current respectively; T_E denotes central-eastern Pacific SST; h is the
 139 objectively-optimized recharge index developed in Izumo and Colin (2022) study (panel b here adapted from
 140 Fig. 2h of this study), i.e. SLA (or Z20) averaged over the equatorial and southwest Pacific (black boxes in
 141 b), namely $h_{ind_{eq+sw}}$ ($h_{ind_{eq+sw}} \approx h_{eq+sw}$, note that the RDO robustness does not depend much of the recharge
 142 index choice; cf. section 2).

143 **2. Data and methods**

144 See Supplementary text S1 for a detailed description of the typical statistical methods
145 we use, plus additional details on data, indices and validation/sensitivity tests. Note that for the
146 non-linear seasonal version of the RDO (section 5), the scripts for obtaining the RDO
147 parameters through multivariate linear regression fits and for running the RDO forced by red
148 noise were developed in python language within the framework of the RO community model
149 under development as a follow-up of the RO review by Vialard et al. (in preparation).

150 *a. Data*

151 For SST observations we use the following monthly datasets: Optimum Interpolation
152 SST OISSTv2 based on *in situ* observations and satellite measurements for the recent period
153 (November 1981-March 2022; Reynolds et al. 2002; hereafter ‘obs1’); HadISSTv1.1 SST
154 (1870-March 2022; Rayner et al. 2003) when using longer-time-scale reanalyses. We also use
155 GPCP monthly precipitation (Adler et al. 2003) for developing the approximated formula of
156 sensitivity of precipitation to Relative SST.

157 We use SLA as a proxy for thermocline depth (e.g. 20°C isotherm depth Z20) and
158 Oceanic Heat Content (OHC) anomalies (e.g. Rebert et al. 1985; Gasparin and Roemmich
159 2017; Palanisamy et al. 2015; see Izumo and Colin 2022 for a detailed comparison of SLA,
160 Z20 and OHC for the recharge index). SLA is advantageous because it is measured since three
161 decades by satellites (Copernicus product; 1993-mid2021; ‘obs1’), allowing a homogeneous
162 spatiotemporal sampling. We also use longer SLA datasets for comparison and to increase the
163 number of effective degrees of freedom: ECMWF ORAS5 oceanic reanalysis extended version
164 (1959-2018; Zuo et al. 2019; hereafter ‘obs2’; we also use its subsurface potential temperature
165 and currents [currents only available from 1975]); even longer SLA from a historical oceanic
166 reanalysis, SODA2.2.6 (1871-2008; Giese and Ray 2011; without subsurface assimilation to
167 avoid spurious trends; hereafter ‘hist’), required to estimate the non-linear seasonal version of
168 the RDO, for which the number of parameters to fit strongly increases (section 5). We subtract
169 from the SLA its 60°S-60°N global average at each time step to remove sea level rise global
170 trend due to global warming, and we also remove any additional regional trend through a linear
171 regression.

172 32 CMIP5 and 47 CMIP6 (Supplementary Table S1; Coupled Model Intercomparison
173 Project phases 5 and 6) models are analysed here. They are the ones for which we have SST
174 and SLA available, for the historical runs (usually 1861-2005 for CMIP5 and 1850-2015 for

175 CMIP6). This amounts to more than 10000 years of model data to test the validity of our
176 hypotheses.

177 *b. ENSO indices*

178 For the ENSO index, T_E , we use the usual Niño3.4 region (170°W-120°W, 5°N-5°S),
179 where the main ENSO SST variability is located in observations. We define T_E as Niño3.4
180 relative SST (RSST, i.e. SST minus its 20°N-20°S tropical mean; Vecchi and Soden 2007), as
181 recommended by Izumo et al. (2020) and Van Oldenborgh et al. (2021) because atmospheric
182 tropical deep convection interannual anomalies are rather related to RSST than to SST, notably
183 under external forcing, i.e. anthropogenic (see also Johnson and Xie 2010; Johnson and Kosaka
184 2016; Williams and Patricola 2018, and Okumura 2019) or volcanic forcing (Khodri et al.
185 2017; Izumo et al. 2018). The deep convection threshold $SST > \sim 27-28^\circ\text{C}$ (e.g. Gadgil et al.
186 1984) translates into $RSST > \sim 1^\circ\text{C}$, a threshold that remains valid with global warming (e.g.
187 Johnson and Xie 2010). See Supplementary Text S3 of Izumo and Colin 2022 for an extensive
188 discussion justifying the relevance of RSST for ENSO and RO equations. Note that using SST
189 instead of RSST gives quasi-similar results. But it is safer to use RSST because of external
190 forcing. T_E is normalized by its standard deviation (STD).

191 For the recharge index (see details and sensitivity tests in Supplementary Text S1), there
192 has been a debate on which recharge index best measures the slow recharge/discharge process:
193 which averaging region (e.g. western Pacific (120°E-155°W) or whole equatorial band (120°E-
194 80°W) h_w or h_{eq} in 5°N-5°S) and which variable (SLA, Z20 (i.e. Warm Water Volume WWV)
195 or OHC) should we use (e.g. Meinen and McPhaden 2000; Planton et al. 2018; Izumo et al.
196 2019) ? Izumo and Colin (2022) have thus developed a physically-unambiguous and
197 objectively-optimized index of the ENSO slow recharge mode, out of phase with T_E (i.e.
198 uncorrelated to T_E at lag 0 and “independent” of the fast adjustment zonal tilt mode, by
199 regressing out from h its component linearly related to T_E approximately considered to be this
200 fast tilt mode). Here we thus use their optimal improved index h_{ind_eq+sw} : the SLA averaged
201 over the equatorial plus southwestern Pacific ($eq+sw$ box: equatorial box 5°N-5°S, 120°E-
202 80°W + southwest box 5°S-15°S, 120°E-170°W), from which T_E -variability has been regressed
203 out, to build a recharge index statistically-independent from T_E . In other words, $h_{ind_eq+sw} =$
204 $h_{eq+sw} - K T_E \approx h_{eq+sw}$ (as regression coefficient K is small thanks to the addition of the South
205 West Pacific region to the equatorial band for the recharge index, e.g. 0.30 for obs2, if all
206 indices are normalized; i.e shared variance between h_{eq+sw} and T_E only of 9%). Its advantage is
207 that it is “orthogonal” to T_E , so it is a pure additional degree of freedom capturing the slow

208 recharge, without being polluted by the fast tilt mode. Also, its approximation h_{eq+sw} is simple
209 and easy to compute.

210 Anyway, we would like to emphasize that the RDO robustness does not depend a lot of
211 the recharge index choice. The RDO implementation would still be useful and robust if one
212 were using usual indices, e.g. h_{eq} or h_w , with β , F_1 and F_2 terms still highly significant (cf.
213 section 4). Furthermore, both β and the final correlation skill of dT_E/dt tendency equation (8)
214 will be mathematically *exactly* the same whether or not we remove the dependent component
215 of h index, as T_E is also one of the other variables of the multivariate regression. Therefore, if
216 the users have a preference, they can also use the typical h_{eq} or h_w indices for the
217 implementation of RDO. Only the overall skill of the RDO equations to represent observed
218 temperature and recharge tendencies will be weaker than if h_{ind_eq+sw} or h_{eq+sw} is used. For the
219 sake of simplicity and clarity of the equations, the optimal recharge index h_{ind_eq+sw} will be
220 hereafter noted as “ h ”.

221

222

223 **3. The two main simple ENSO conceptual models, the RO and the DO**

224 *a. The Recharge Oscillator (RO)*

225 The RO theory (Jin 1997ab) focuses on the slow oceanic negative feedback on SST
226 associated with wind-driven equatorial OHC variations (Meinen and McPhaden 2000), as
227 aforementioned. It brings long oceanic memory across ENSO phases. During a La Niña,
228 easterlies favor a slow recharge of OHC (i.e. a deepening of the thermocline depth h) in the
229 equatorial and southwestern Pacific (Fig. 1b). The El Niño event will in turn lead to a discharge
230 favoring a reversal to La Niña conditions, and so on.

231 The RO equations (Jin et al. 2020; Izumo and Colin 2022) are:

232

$$233 \quad dT_E/dt = R_{RO} T_E + F_{I_RO} h \quad (4)$$

$$234 \quad dh/dt = -F_2 T_E - \varepsilon h \quad (5)$$

235

236 where R_{RO} is the net effect of Bjerknes positive feedback and instantaneous negative
237 feedback, F_{I_RO} the recharge/discharge influence on T_E , F_2 the slow recharge/discharge forced
238 by La Niña/El Niño and ε a Newtonian damping on h . The subscript RO is added to avoid
239 ambiguity when necessary.

240 Using the improved optimal recharge index $h=h_{ind_eq+sw}$, independent of T_E (cf. section 2),
 241 ε is negligible in observations and in all CMIP models (Fig. 4b, see section 4.b). Therefore, we
 242 neglect it at first for the simplest RDO framework (linear and without seasonal cycle). This RO
 243 system of two differential equations has the form of a harmonic oscillator (Jin 1997a).

244 *b. The Delayed Oscillator (DO)*

245 The zonal advective-reflective delayed oscillator (DO) is based on the relatively rapid
 246 equatorial wave reflections causing a 4 to 8-month delayed negative feedback dominated by
 247 the zonal advective term (Picaut et al. 1997; Vialard et al. 1998) in the central Pacific where
 248 the coupling with atmospheric deep convection is the largest. E.g. El Niño westerlies force an
 249 equatorial downwelling Kelvin wave $K_{d,forced}$ (i.e. equatorial zonal current anomaly $U' > 0$ and
 250 eastward displacement of the warmpool eastern edge) reflecting at the eastern boundary into a
 251 downwelling Rossby wave $R_{d,reflection}$ (i.e. $U' < 0$) bringing back the edge westward half a year
 252 later (Fig. 1a) (there is also a $R_{up,forced}$ forced to the west of the westerly anomaly, propagating
 253 westward and reflecting at the western boundary as an eastward $K_{up,reflection}$ wave; e.g.
 254 Boulanger and Menkes 2001). This strong zonal advective delayed feedback favoring ENSO
 255 phase reversal is well observed, simulated and understood. Formally, it relates zonal current
 256 interannual anomalies to the evolution of zonal wind stress anomalies τ_x over the preceding
 257 months. The equatorial Pacific Ocean dynamics being quasi-linear (at intraseasonal to
 258 interannual timescales), the response to any windstress anomaly $\tau_x(t)$ with a typical ENSO wind
 259 pattern is the convolution of this $\tau_x(t)$ to the linear oceanic impulse response. Thus, the
 260 anomalies can be approximated as a “causal” filter of τ_x : $\tau_x^{Am}(t-t_1) - c \tau_x^{Bm}(t-t_2)$ where t_1 is ~ 0
 261 to 2 months, the difference $t_2 - t_1$ is ~ 4 to 6 months, where the superscript, e.g. Am , stands for a
 262 A -month running mean and c varies depending on τ_x ' anomaly location, and of the variable
 263 (either U , SLA or SST; cf. Izumo et al. 2016). For U_C anomalies, they are approximately
 264 proportional to the zonal wind stress anomalies 1–2 months earlier minus a fraction of the wind
 265 stress anomalies 7–8 months earlier. The first term represents the fast oceanic response, while
 266 the second one represents the delayed negative feedback associated with wave reflection at
 267 both boundaries. I.e. a realistic approximation of U_C is:

268
 269
$$U_C(t) \approx A (\tau_x(t) - \beta_0 \tau_x(t-\eta)) \quad (6)$$

270
 271 where τ_x is equatorial zonal windstress, A is a constant and β_0 is e.g. $\sim 2/3$ for U_C at $0^\circ N$,
 272 $170^\circ W$ (based on Table 1 of Izumo et al. 2016 combined with the approximation $\tau_x^{2m} \approx \tau_x^{1m}$

273 valid for periods longer than intraseasonal). We consider the delay η to be approximately 6
 274 months for the sake of simplicity, as it is rather well constrained by equatorial wave propagation
 275 times in a fixed basin geometry, with the main mode (first baroclinic mode) celerity well known
 276 in the equatorial Pacific. This formula is an excellent approximation of the response of the
 277 quasi-linear continuously-stratified equatorial Pacific Ocean (with a realistic coastline) to a
 278 windstress forcing with an ENSO-like spatial pattern. It is more realistic than previous
 279 approximations of U_C used in previous RO-based conceptual models.

280 We can combine (6) with some typical assumptions to derive a DO equation for T_C , where
 281 T_C is the SST anomaly in the central equatorial Pacific. We suppose: 1) $\tau_x \approx \mu_C T_C$ (μ_C being a
 282 coupling parameter), 2) the zonal SST gradient in the central Pacific to be its climatological
 283 value (i.e. independent of the anomaly T_C) at 1st order ($dT_C/dx \approx dT_{C_clim}/dx \approx -C$, with $C > 0$;
 284 hence the zonal advection term is CU_C ; relaxing this assumption leads to non-linear terms such
 285 as in section 5), and 3) that atmospheric fluxes can be approximated as a weak Newtonian
 286 damping (term $-r_{damp}T_C$), weaker than the Bjerknes feedback related to the 1st term in the above
 287 equation for U_C . Thus, we have, using (6):

288

$$289 \quad dT_C/dt \approx -U_C(t) dT_C/dx(t) - r_{damp} T_C(t) \approx AC\mu_C (T_C(t) - \beta_0 T_C(t-\eta)) - r_{damp} T_C(t) \quad (7.0)$$

290

291 We obtain the usual DO equation:

292

$$293 \quad dT_C/dt = R_{DO} T_C(t) - \beta_{DO} T_C(t-\eta) \quad (7)$$

294

295 where $R_{DO} = AC\mu_C - r_{damp}$ and $\beta_{DO} = A\beta_0C\mu_C$ are positive constants. From here onwards,
 296 we skip the notation (t) when the variable is taken at time t without delay.

297
 298

299 **4. Building the RDO model by merging the RO and DO models**

300 *a. Combining RO and DO equations*

301 The RO captures the long-term recharge of h influencing the SST in the central-eastern
 302 Pacific. The DO captures the faster delayed advective-reflective negative feedback influencing
 303 the SST in the western-central Pacific. Knowing the importance of central Pacific SST for
 304 convection and thus zonal wind, and that the negative feedbacks involved in the DO and RO

305 are physically different, it thus seems relevant to keep these different feedbacks as two distinct
 306 processes combined in a hybrid « Recharge Delayed Oscillator » (RDO), whose set of
 307 equations is:

$$308 \quad \frac{dT_E}{dt} = R T_E + F_1 h - \beta T_E(t-\eta) \quad (8)$$

$$309 \quad \frac{dh}{dt} = -F_2 T_E - \varepsilon h \quad (9)$$

311
 312 Equation (8) merges (4) and (7); (9) is as equation (5). Here central-eastern Pacific T_E and
 313 central Pacific T_C are merged into one variable representing a single region, the central-eastern
 314 Pacific, hereafter T_E (Niño3.4 RSST; cf. section 2), for the sake of simplicity. Even though we
 315 replace T_C by T_E , we still consider η to be about 6 months, because physically the zonal
 316 advection delayed feedback operates more on temperature “weighted” by atmospheric
 317 convection sensitivity (cf. below), i.e. more in the central Pacific than in the eastern Pacific
 318 (while the recharge thermocline feedback influences more the central-eastern Pacific; cf.
 319 following section).

320 This RDO simple set of equations sounds conceptually compelling. Notably it does not
 321 require the low-frequency approximation (used in Fedorov 2010 and Clarke 2010) to be valid,
 322 as the RDO takes into account the two main “fast” wave processes, the fast mode adjustment,
 323 and the zonal-advective delayed negative feedback.

324 *b. Confirming the RDO through observations/reanalyses*

325 We now want to verify empirically that this RDO set of equations is justified and
 326 statistically grounded, in observations as well as in the 79 CMIP models.

327 First, we verify that separating the zonal-advective delayed feedback and the recharge
 328 process through equation (8) makes sense. I.e. that $+F_1 h$ and $-\beta T_E(t-\eta)$ represent two distinct
 329 physical processes. We do a multivariate linear regression for an equation similar to equation
 330 (8), but for the tendency of temperature $T(x,y,t)$ anywhere in the tropical Pacific:

$$331 \quad \frac{\partial T(x,y,t)}{\partial t} = R(x,y) T_E + F_1(x,y) h - \beta(x,y) T_E(t-\eta) \quad (8.1)$$

332
 333
 334 The regression quantifies the contributions of $-\beta(x,y)T_E(t-\eta)$ and $F_1(x,y)h$ terms to $T(x,y,t)$
 335 tendency at each grid point. Fig. 2ab compares their spatial patterns in the tropical Pacific, that
 336 are indeed quite different (orthogonal). Indeed, we see that the $-\beta T_E(t-\eta)$ contribution is
 337 stronger in the western-central Pacific, especially near the warm pool eastern edge, where the

338 zonal advection term dominates. It confirms that $-\beta T_E(t-\eta)$ represents the zonal advective-
339 reflective delayed negative feedback. Conversely, the recharge term $F_I h$ is larger in the eastern
340 Pacific as expected, as the thermocline feedback term is stronger there (i.e. the
341 recharge/discharge process is more important for EP than for CP El Niño events; e.g. Hasegawa
342 et al. 2006; Kug et al. 2009, 2010; McPhaden 2012; Capotondi 2013; Ren and Jin 2013). Fig.
343 3a compares more quantitatively their values along the equator. The contribution of $-\beta T_E(t-\eta)$
344 is larger than that of $F_I h$ in the western-central Pacific (around Niño4 region), while the
345 contribution of $F_I h$ is the largest in the central eastern region (from $\sim 150^\circ\text{W}$, the western
346 boundary of Niño3 region), for both obs1 and obs2 datasets (that are consistent, with
347 differences not significant at the 90% level, cf. Suppl. Fig. S1; while β and F_I differences are
348 statistically significant). Note that $R(x,y,t)$ is, as β , larger in \sim Niño4 region (Suppl. Fig. S1d,
349 likely because of the largest positive Bjerknes feedback there). R is however only about one
350 third of β there. The correlation skill of equation (8.1) is high east of the dateline, being at its
351 largest around Niño3.4 region (Suppl. Fig. S1d), where β and F_I roles add up. This also
352 confirms that focusing on Niño3.4 region is a good approximation.

353 Considering only RSST tendency (Fig. 2ab), we could think that the recharge term
354 dominates the delayed term. Yet, in terms of feedback to deep
355 convection/precipitation/windstress, the deep convection (the primary contributor to
356 precipitation in the Tropics) sensitivity to RSST anomalies is larger in the western Pacific
357 warmpool region, where climatological RSST is larger, than in the central-eastern cold tongue
358 region (e.g. He et al. 2018 study had estimated a formula for the precipitation sensitivity to
359 SST, $P(SST)$). To illustrate this, we can use a simple exponential formula we have developed
360 as an approximation of the precipitation sensitivity to local RSST, $P(RSST)$ (Appendix Fig.
361 A1):

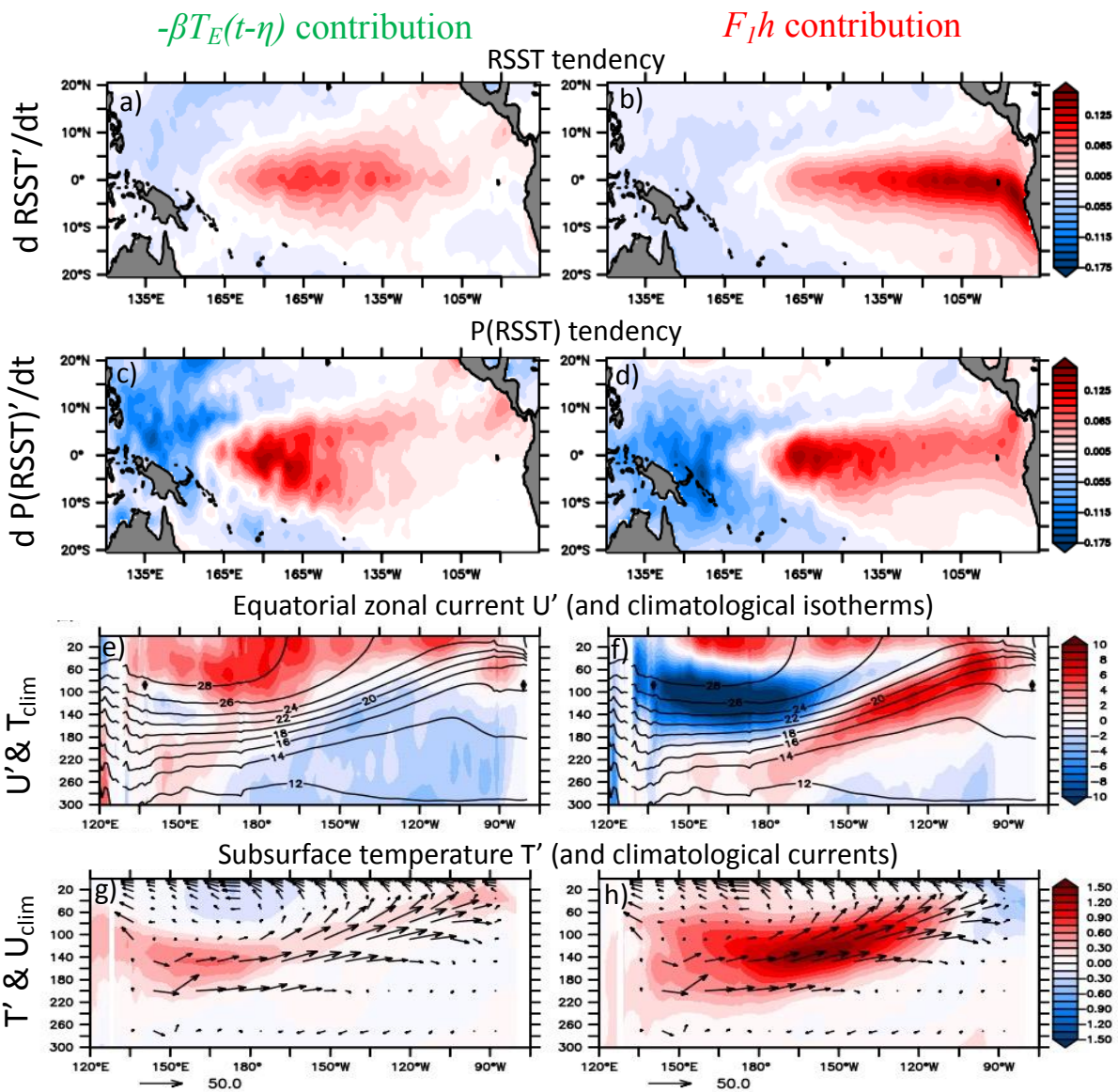
$$362$$

$$363 \quad P(RSST) = 2 e^{0.56RSST} \quad (8.2)$$

$$364$$

365 Where RSST is the total RSST field (including both climatology and anomaly, i.e.
366 $RSST_{\text{clim}} + RSST'$). The constant 0.56 is taken from the term representing the effect of RSST
367 on deep convection in Tippett et al. (2011) cyclogenesis index (efficient at describing the
368 cyclogenesis seasonal cycle at the global scale; Menkes et al. 2012). The factor 2 (in mm day^{-1})
369 is a crude fit to the scatterplot of $Precip_{\text{clim}}$ to $RSST_{\text{clim}}$ in the equatorial Pacific (this factor
370 will anyway not influence the relative contributions to $dP(RSST)/dt$ of $-\beta T_E(t-\eta)$ and $F_I h$ that
371 are our focus here). Fig. 2cd is as Fig. 2ab, but for $dP(RSST)/dt$. The contribution to convective

372 precipitation tendency of $-\beta T_E(t-\eta)$ has actually, in the western-central Pacific, an almost
 373 similar amplitude to the contribution of $F_I h$. Note that these estimated contributions to local
 374 precipitation will be then amplified by the convergence feedback (Conditional Instability of
 375 the Second Kind; CISK), notably for strong El Niño events in the central-eastern Pacific
 376 (Srinivas et al. 2022). To conclude, the $-\beta T_E(t-\eta)$ term can force large tendencies in precipitation
 377 and related equatorial zonal windstress, crucial for the ocean-atmosphere coupling. Hence
 378 adding the $-\beta T_E(t-\eta)$ term to the $\partial T(x,y,t)/\partial t$ equation can make the conceptual model
 379 significantly more realistic.



380

381 **Fig. 2. Distinct physical processes represented by $-\beta T_E(t-\eta)$ and $F_I h$ terms in RDO, and related ENSO**
 382 **spatiotemporal diversity captured by the RDO. a and b) contributions of respectively $-\beta(x,y)T_E(t-\eta)$ and**
 383 **$F_I(x,y)h(t)$ to $\partial T(x,y,t)/\partial t(t)$ in Equation 8.1 (i.e. the regression coefficients $\beta(x,y)$ and $F_I(x,y)$ for normalized**
 384 **T_E and h , i.e. unit in $^{\circ}\text{C}\cdot\text{month}^{-1}$ per STD of T_E or h) for obs2. By reconstructing $T(x,y,t)$ from these patterns,**
 385 **the RDO can simulate part of ENSO spatial and spectral diversity (cf. section 4f). c and d) as a and b, but**
 386 **for the expected response of deep convection/precipitation to RSST tendency shown in a and b (mm day^{-1})**

387 month⁻¹), more sensitive to RSST anomalies in the western Pacific warmpool than in the central-eastern cold
 388 tongue region (based on equation 8.2: $P(RSST) = 2 e^{0.56RSST}$). Lower panels show the oceanic mechanisms
 389 behind $-\beta T_E(t-\eta)$ and $F_I h$ contributions to $\partial T(x,y,t)/\partial t$. **e** and **f** are as **a** and **b**, but for equatorial (2°N-2°S
 390 average) subsurface zonal current anomaly U' (color, cm/s), with climatological isotherms T_{clim} overlaid
 391 (black contours, °C). **g** and **h** are as **a** and **b**, but for subsurface potential temperature anomaly T' (color, °C),
 392 with climatological zonal and vertical currents U_{clim} and W_{clim} overlaid (black vectors, cm s⁻¹).
 393

394 To understand the oceanic mechanisms behind $-\beta(x,y)T_E(t-\eta)$ and $F_I(x,y)h(t)$ contributions
 395 to $\partial T(x,y,t)/\partial t$, remember that the temperature tendency of the oceanic mixed layer is firstly due
 396 to heat advection (see its full decomposition in Supplementary Figures S2 an), with surface
 397 fluxes acting to damp interannual SST anomalies. So as to understand heat advection
 398 anomalies, Fig. 2 lower panels show equatorial subsurface zonal current anomaly U' , with
 399 climatological isotherms T_{clim} overlaid, as well as subsurface potential temperature anomaly
 400 T' , with climatological currents U_{clim} and W_{clim} overlaid.

401 The $-\beta T_E(t-\eta)$ contribution is firstly through a large positive U' anomaly in the upper
 402 layer. U' is maximal near the dateline (up to 6 cm s⁻¹ per STD of $T_E(t-\eta)$; Fig. 2e), consistent
 403 with the timing of the westward propagation of the equatorial upwelling Rossby wave forced
 404 by easterly anomalies 6 months earlier (in the case of a prior La Niña with negative T_E and
 405 easterlies; Fig. 1a was shown for the opposite westerly case). This eastward current anomaly
 406 U' advects the negative climatological zonal temperature gradient $\partial T_{clim}/\partial x$, i.e. $-U' \partial T_{clim}/\partial x$
 407 (Suppl. Fig. S3c), favoring a warming of the oceanic mixed layer in the central Pacific (of up
 408 to 0.15K month⁻¹, i.e. about 1K in half a year, per STD of $T_E(t-\eta)$; Suppl. Fig. S2c). The T'
 409 anomalies related to $-\beta T_E(t-\eta)$ are conversely weak near the surface (about 0.2K per STD of
 410 $T_E(t-\eta)$; Fig. 2g) and have a secondary role (Suppl. Fig. S3a) in the heat zonal advection of the
 411 mixed layer (and at the subsurface, they are much weaker than those related to $F_I h$; see also
 412 Fig. 3c). This analysis confirms that $-\beta T_E(t-\eta)$ contribution is firstly through zonal advection
 413 in the central Pacific, i.e. $-\beta T_E(t-\eta)$ represents the zonal advective-reflective delayed negative
 414 feedback.

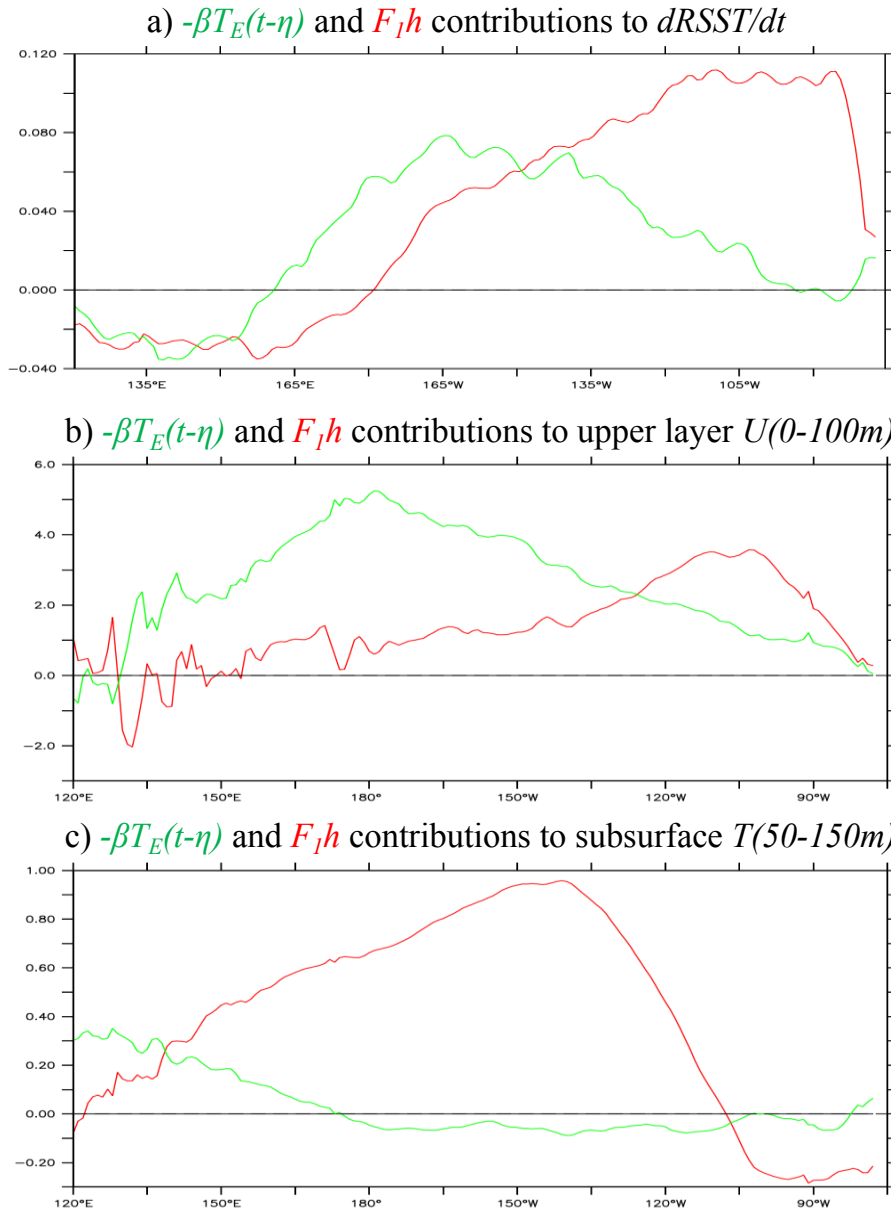
415 The recharge influence $+F_I h$ represents a different physical process. $F_I h$ influence is
 416 through warmer EUC and upwelling because of the warm subsurface anomaly in the central-
 417 eastern Pacific due to the recharge and related deeper thermocline. h recharge favors a large
 418 subsurface warm anomaly in the heart of the Equatorial UnderCurrent (EUC shallower in the
 419 east, like the thermocline, from ~150m at 180°E to ~100m at 130°W). This warm T_{EUC}'
 420 anomaly, partly due to the deeper thermocline (cf. Izumo 2003, 2005), will be advected by the
 421 climatological EUC, i.e. a positive term $-U_{clim} \partial T'/\partial x$ (Suppl. Fig. S3b). It is also within the
 422 climatological equatorial upwelling, i.e. $-W_{clim} \partial T'/\partial z$ (Suppl. Fig. S3f). I.e. the EUC will bring

423 warmer waters to the central-eastern Pacific, where they will be upwelled to the mixed layer
424 by the climatological upwelling, favoring an overall warming tendency of the mixed layer of
425 the central-eastern equatorial Pacific (Suppl. Fig. S2b). Note that there are secondary roles of:
426 1) an increase of the deepest part of the EUC in the east (because of increased zonal gradient
427 of thermocline since the recharge is more in the western-central than in the eastern Pacific), i.e.
428 $-U' \partial T_{clim}/\partial x$ (Suppl. Fig. S3d), 2) an upwelling reduction in the extreme east, east of $\sim 100^\circ\text{W}$,
429 i.e. $-W' \partial T_{clim}/\partial z$ (Suppl. Fig. S3h).

430 To better understand the origin of this warm subsurface anomaly, we have to remember
431 that the EUC is part of a large-scale 3D circulation. It is fed by the north and south meridional
432 pycnocline convergences, at their largest in the western Pacific. They are the lower branches
433 of the shallow SubTropical/Tropical Cells (STCs/TCs; McCreary et al. 1992; McCreary and
434 Lu 1994; Zeller et al. 2019, 2021). The south convergence is climatologically two times larger
435 than the north one (because of the potential vorticity barrier in the north) and is the first origin
436 of EUC waters (Blanke and Raynaud 1997; Stellingma et al. 2022). Hence, the h recharge
437 associated to deeper thermocline in the equatorial and southwest Pacific favors warmer
438 transport-weighted temperatures of the meridional pycnocline convergences, notably the
439 dominant south convergence related to southwest Pacific recharge, thereby feeding the EUC
440 with warmer waters (Izumo 2003, 2005; Ballester et al. 2015, 2016). Note by the way that the
441 north-south asymmetry in the convergence further justifies the use of the asymmetric recharge
442 index h_{eq+sw} .

443 Fig. 3bc summarizes the distinct dominant oceanic processes for the delayed feedback
444 (green line) and recharge term (red). The delayed feedback dominates for U' in the west and
445 central Pacific upper layer, i.e. for zonal advection $-U' \partial T_{clim}/\partial x$. The recharge term dominates
446 for the subsurface temperature anomaly T' advected by the mean 3D circulation, i.e. by the
447 climatological EUC through $-U_{clim} \partial T'/\partial x$, and by climatological upwelling through $-W_{clim}$
448 $\partial T'/\partial z$, in most of the equatorial Pacific.

449



450

451 **Fig. 3. Equatorial contributions of delayed feedback $-\beta T_E(t-\eta)$ (green) and of recharge term $F_I h$ (red).**

452 **a)** as Fig. 2ab for RSST tendency, but for the 5°N-5°S average. **b)** as a but for upper layer (0-100m average)

453 zonal current. **c)** as a but for subsurface potential temperature (50-150m average).

454

455 Overall, these analyses confirm from observations/reanalysis that this RDO is physically-

456 grounded, the two terms $-\beta T_E(t-\eta)$ and $F_I h$ being both important and representing distinct

457 physical processes. It is hence more physical to distinguish these two terms, equation (8) being

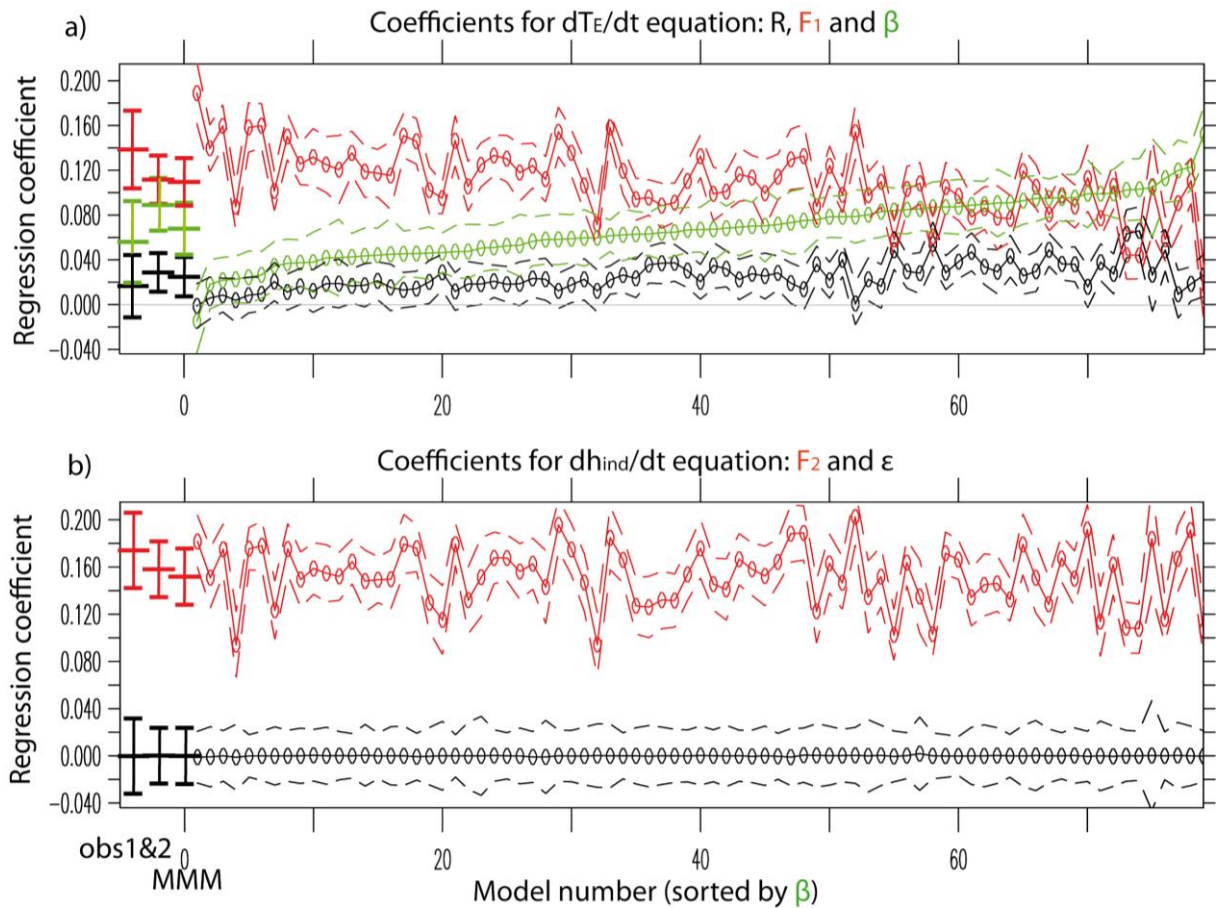
458 one reasonable way to conceptualize them. Furthermore, we will later on show that the different

459 spatial patterns of β and F_I seen in Fig. 2ab allow the RDO to capture partly the ENSO

460 spatiotemporal diversity in equatorial Pacific $T(x,y,t)$. But for now, we will firstly analyze the

461 RDO in its simplest form, focusing on spatially-averaged Niño3.4 RSST, namely $T_E(t)$.

462



463

464 **Fig. 4. Significance of the RDO coefficients in observations and CMIP. a)** coefficients R (black), F_1 (red)
 465 and β (green) of the regression of dT_E/dt onto normalized $T_E(t)$, $h(t)$ and $T_E(t-\eta)$ (cf. Equation 8) for the 79
 466 CMIP models sorted by β (numbers 1 to 79 on x-axis). **b)** coefficients F_2 (red) and ϵ (black) of the regression
 467 of dh/dt onto $h(t)$ and $T_E(t)$ (cf. Equation 5). Three thick error bars from the left respectively show obs1,
 468 obs2 and Multi-Model Mean (MME). T_E and h are normalized so that regression coefficients (in month⁻¹)
 469 can be compared, whatever the model's ENSO amplitude. 90% confidence intervals are shown as dashes,
 470 and for observations and MMM as marks.

471

472 A 2nd way of confirming the relevance of the RDO equation (8) is to compute the
 473 coefficients R , F_1 and β obtained from the multivariate linear regression of dT_E/dt onto $T_E(t)$,
 474 $h(t)$ and $T_E(t-\eta)$ for observations (and in the following section for CMIP models), now focusing
 475 on the tendency of spatially-averaged T_E , as shown in the very left of Fig. 4a for observations.
 476 The most striking result is that F_1 and β are *both* highly-significantly positive in obs1 and obs2
 477 (at the 95% level). Therefore, both need to be taken into account, and cannot be neglected in
 478 Equation (8). Consistently, observed h and $T_E(t-\eta)$ have more than half (~60%) of unshared
 479 variance, in agreement with the fact that they firstly represent different physical processes.

480

481 *c. Confirming the RDO through CMIP multi-model database; mean state influence*

482

483 Accordingly, the RDO equation is also relevant for most CMIP models, with F_1 and β
484 being both significantly positive in almost all the CMIP models (Fig. 4). In the following, we
485 write variable $X = X1[X2]\{X3\}$ with X1 for values obtained with ‘obs1’ (OISST and satellite
486 SLA), X2 for ‘obs2’ (HadiSST and ORAS5 SLA) and X3 for Multi Model Mean (MMM) of
487 CMIP5 and CMIP6 historical runs. CMIP MMM values lie within observational uncertainties:
488 $(F_1, \beta) = (0.14 \pm 0.03, 0.06 \pm 0.04) [0.11 \pm 0.02, 0.09 \pm 0.02]$ month⁻¹ for observations and
489 $\{0.11 \pm 0.02, 0.07 \pm 0.02\}$ for CMIP (similar values for CMIP5 and CMIP6 MMM). Note that the
490 relative strengths of F_1 and β can also depend of the period used (consistent with the fact that
491 the dominant ENSO flavor may have changed with time). E.g. longer obs2 can be compared to
492 obs1 on their 26-yr long overlapping period, 1993-2018. It gives $(F_1, \beta) =$
493 $[0.13 \pm 0.02, 0.06 \pm 0.02]$ month⁻¹, almost similar to obs1, with F_1 larger than β . While obs2 on
494 the 26-yr earlier period 1959-1984 gives $(F_1, \beta) = [0.09 \pm 0.02, 0.10 \pm 0.02]$ month⁻¹, with
495 conversely F_1 weaker than β (possibly because of higher-frequency ENSO during that period;
496 cf. section 4; such decadal changes could partly explain the changes in observed WWV/ T_E lag-
497 relationship shown by McPhaden 2012 and confirmed by Crespo et al. 2022). The R coefficient
498 is weaker, $R = 0.02 \pm 0.03 [0.03 \pm 0.02]\{0.02 \pm 0.02\}$ month⁻¹, but is still significantly positive at
499 the 90% level in the majority of CMIP models in this new RDO framework, contrary to R_{RO}
500 that is not significantly different from zero in the pure RO framework. The correlation skill of
501 the RDO dT_E/dt equation (8) is highly significant ($r_{T_e} = 0.71 [0.68]\{0.64\}$), also confirming the
502 RDO relevance. The Steiger’s test (on the difference between these two dependent correlations
503 from a single sample; note that the correlations are overlapping, sharing one variable in
504 common) allows us to compute the significance of the improvement of the skill (and of its
505 square, i.e. explained variance) in obs1[obs2]: from RO to RDO, this significance is 84%[97%]
506 (explained variance increases from 48% to 51% [41% to 46%]), and from DO to RDO, it is
507 99%[99%] (explained variance increases from 34% to 51% [35% to 46%]). For CMIP MMM,
508 the improvement is even more significant statistically, given the much longer timeseries: the
509 explained variance increases by ~10% from RO to RDO (from 34 to 38%) and by ~20% from
510 DO to RDO (from 29% to 38%). Note that, in the most realistic version of the RDO, the non-
511 linear seasonal RDO developed later, the improvement is even much larger (section 5).

512 Interestingly, there is a large CMIP inter-model diversity in the relative strengths of the
513 coefficients F_1 and β , larger than for R (Fig. 4a). The models (sorted by the amplitude of β in
514 Fig. 4) that have a large F_1 tend to have a weak β , and vice versa (inter-model correlation = -

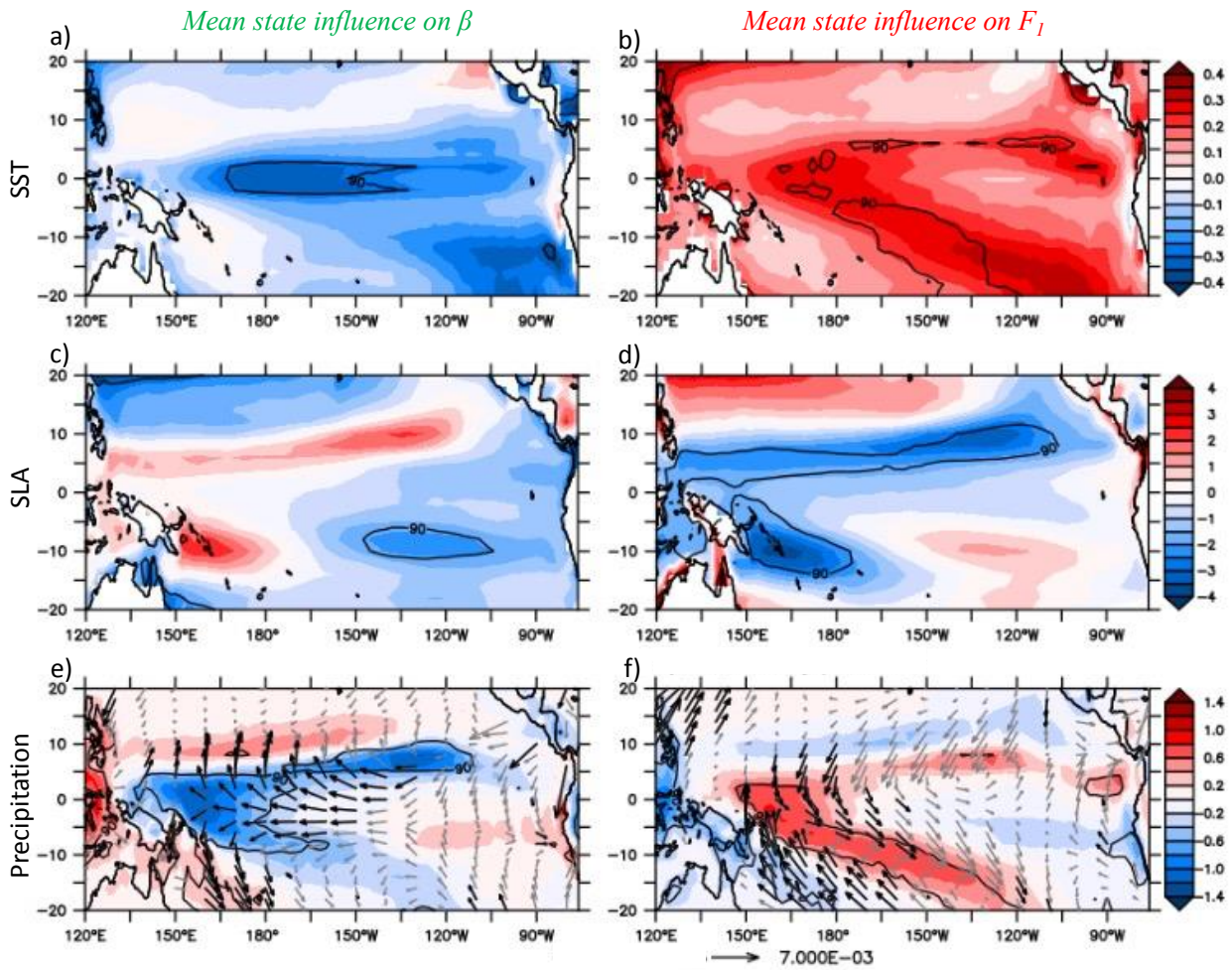
515 0.69). R is positively correlated to β (0.59) and negatively correlated to F_1 (-0.75). This large
516 diversity of the parameters is likely related to the diversity of spectral, spatial and amplitude
517 (related to growth rate) characteristics of ENSO and of Pacific mean state (cf. below) among
518 CMIP models.

519 Fig. 4b similarly shows the coefficients F_2 and ε for the regression of dh/dt onto $T_E(t)$ and
520 $h(t)$. F_2 is highly significant in all observations and models, $F_2 = 0.17 \pm 0.03$
521 $[0.16 \pm 0.02] \{0.15 \pm 0.02\}$ month⁻¹ and more consistent in amplitude among CMIP than F_1 and
522 β , with a weaker inter-model diversity ($\sim \pm 30\%$ of MMM value) positively correlated with F_1
523 (inter-model correlation = 0.79). As in observations, ε is negligible in all CMIP models
524 (confirming Izumo and Colin 2022 result for CMIP models: using an independent index such
525 as “ h ”= h_{ind_eq+sw} rather than other recharge indices polluted by the tilt mode component leads
526 to a negligible ε ; note that we expect oceanic dissipation to be weak, e.g. Fedorov 2010). The
527 correlation skill of the regression is again highly significant: $r_h = 0.67 [0.67] \{0.57\}$.

528 CMIP MMM is qualitatively similar to observations, but each model has its own biases.
529 One may hence question how sensitive these results are to ENSO realism in CMIP models. So
530 we have evaluated ENSO in each CMIP model by its ENSO amplitude ($STD(T_E)$) and spectral
531 shape (namely the ratio between the spectral density in the 1–3 years band and that in 3–15
532 years band, i.e. $STD(1/3-1/1 \text{ yr}^{-1} \text{ band-pass filtered Ni\~{n}o3.4 RSST})$ divided by $STD(1/15-1/3$
533 $\text{ yr}^{-1} \text{ band-pass filtered Ni\~{n}o3.4 RSST})$, close to Bellenger et al. (2014) metrics; more detailed
534 evaluations of CMIP ENSO can be found in earlier references; e.g. Bellenger et al. 2013). We
535 have found some models that are quite far from observations for these two metrics, especially
536 in CMIP5. We have then selected the most realistic CMIP models and have redone Fig. 4 for
537 them (Suppl. Fig. S4). Consistently, the models with the lowest β values (on the left) are
538 rejected, as well as most models with the largest β values. But anyway, the MMM of the RDO
539 parameters are almost similar (to the MMM of all models without selection) for this set of
540 selected models (as well as for CMIP5 and CMIP6 separately). These verifications confirm
541 that the results here are robust, β becoming even more statistically significant when selecting
542 the most realistic models.

543 The next step is to understand what causes the inter-model diversity in RDO parameters.
544 Knowing that β and F_1 represent distinct physical processes, we expect their strengths to have
545 distinct dependances to the mean state of the climate model. They should depend notably on
546 the SST zonal gradient, the position of the warm pool eastern edge, the mean thermocline depth,
547 themselves possibly related to the typical cold tongue bias, or to the western Pacific
548 precipitation bias (e.g. Bayr et al. 2018, 2019a) with double ITCZ (InterTropical Convergence

549 Zone) bias and related SPCZ (South Pacific Convergence Zone) bias. We can explore this in
550 e.g. CMIP5 models. Fig. 5 shows the inter-model regressions of model mean state (in terms of
551 SST, SLA, precipitation and windstress) onto β and F_1 separately. Note that the patterns related
552 to β and F_1 might seem opposite to each other at first glance but they are actually quite different,
553 not simply opposite in sign. β is as expected larger if the cold tongue is stronger in a model
554 (Fig. 5a), i.e. if the negative mean zonal SST gradient in the western-central equatorial Pacific
555 is stronger (Bayr et al. 2018). The stronger cold tongue there is itself related to larger equatorial
556 easterlies related to larger precipitation over the maritime continent and weaker precipitation
557 in the central Pacific with a horseshoe pattern (Fig. 5e). F_1 strength is more correlated to
558 SLA/thermocline depth mean state (Fig. 5d). A possible explanation is the following: the
559 shallower the mean thermocline is in the southwestern and equatorial Pacific (and also at the
560 southern edge of the north ITCZ, where the North Equatorial Counter Current (NECC) is
561 located), the shallower the STCs lower branches are, the more their transport-weighted
562 temperatures can be affected by thermocline depth anomalies related to a recharged or
563 discharged state. The SLA/thermocline depth mean state changes in Fig. 5d are interestingly
564 related not only to equatorial zonal windstress and equatorial cold tongue, but also to
565 meridionally-asymmetric off-equatorial windstress and related curl (with notably Ekman
566 pumping in the SPCZ forcing off-equatorial Rossby waves propagating westward in the
567 southwest Pacific) associated with mean precipitation (and SST) asymmetric changes (Fig. 5f).
568 To sum up, the strength of the delayed feedback and the effectiveness of the recharge/discharge
569 process vary as a result of the CMIP mean state diversity.
570



571

572 **Fig. 5. Mean state influence on β (left) and F_1 (right) in CMIP5 models.** Inter-model regressions, onto β
 573 and F_1 separately, of model mean state, in terms of SST (upper panels; K), SLA (middle panels, with SLA
 574 global mean removed; cm) and precipitation (lower panels; mm day⁻¹; wind stress overlaid in grey, and in
 575 black when significant at the 90% level, vectors direction adjusted to panels aspect ratio; N m⁻²; 90%
 576 significant level overlaid as black contours; two-tailed student t-test). Amplitudes per inter-model STD of β
 577 or F_1 .

578

579 For simplicity, we choose the default values $F_1 = 0.12$, $\beta = 0.08$, $R = 0.02$, $F_2 = 0.16$ and ϵ
 580 $= 0$ month⁻¹, as a simple example to represent their typical values. With these default
 581 parameters, the delayed term contribution is two thirds of the recharge term contribution in the
 582 temperature tendency equation.

583 *d. RDO eigenvalues and their dependency to parameters*

584

585 The solutions of the RDO system of differential equations are a vector space, actually of
 586 infinite dimension because of the delayed term (see details later on and see also Jin 1997a). To
 587 understand the RDO behavior and dependency to parameters, we obtain the eigenvalues by

588 considering complex solutions of the form $e^{\sigma t}$, with $\sigma = \sigma_r + i \sigma_i$, its real part σ_r being the
 589 exponential growth or damping rate, and its imaginary part σ_i being the angular frequency. The
 590 set of equations (8) and (9) (with ε here neglected for the sake of simplicity; including it is
 591 straightforward but makes the below equations heavier; not shown) leads to the following
 592 eigenvalues characteristic equation:

593

$$594 \quad \sigma^2 - R\sigma + \beta\sigma e^{-\sigma\eta} + F_1F_2 = 0 \quad (10)$$

595

596 This equation in the complex space is equivalent to two real equations:

597

$$598 \quad \sigma_r^2 - \sigma_i^2 - R\sigma_r + \beta e^{-(\sigma_r\eta)} (\sigma_r \cos(\sigma_i\eta) + \sigma_i \sin(\sigma_i\eta)) + F_1F_2 = 0 \quad (\text{real part of (10)})$$

$$599 \quad 2\sigma_r\sigma_i - R\sigma_i + \beta e^{-(\sigma_r\eta)} (\sigma_i \cos(\sigma_i\eta) - \sigma_r \sin(\sigma_i\eta)) = 0 \quad (\text{imaginary part of (10)})$$

600

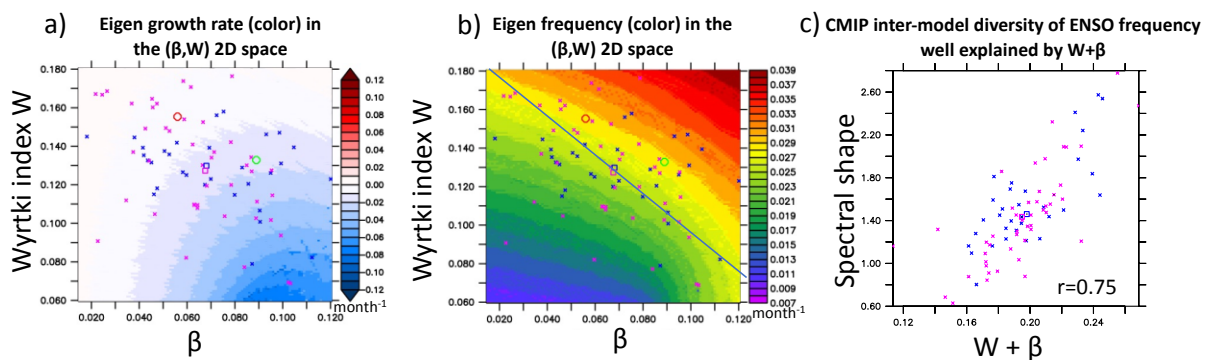
601 The solutions can be found numerically as the intersections of the solutions of each of
 602 these last two equations in the (σ_r, σ_i) space (Suppl. Fig. S5). The pure DO has an infinity of
 603 solutions, while the pure RO has only one. Therefore, the RDO eigenvalues equation (10) has
 604 an infinity of solutions and the model has an infinity of eigenmodes. Around the default values
 605 of the parameters, the 1st mode is a weakly-damped low-frequency mode (close to observed
 606 ENSO main period, and with characteristics relatively close to the RO single mode), with a ~3-
 607 yr period and a ~0.02 month⁻¹ damping rate (i.e. ~1/4 yr⁻¹). The 2nd mode is a highly-damped
 608 higher-frequency mode (~5-month period and ~0.45 month⁻¹ damping rate). This 2nd mode can
 609 add some high-frequency behavior, making the RDO more appealing than the pure RO which
 610 only has one eigenmode close to the 1st RDO mode. Higher eigenmodes are even more damped
 611 and of higher frequency.

612 Eigenvalues equation (10) depends on parameters R , β and on the product F_1F_2 , i.e. on the
 613 Wyrтки angular frequency $W=(F_1F_2)^{1/2}$, on which the eigenfrequency depends in the RO
 614 framework (cf. Lu et al. 2018, Jin et al. 2020). Suppl. Fig. S6def shows the dependency of
 615 RDO eigenfrequency $f=\sigma_i/(2\pi)$ to W , β and R separately. Ranges of W , β and R are chosen by
 616 considering the observations uncertainties and the inter-model diversity in CMIP data shown
 617 in Fig. 4a. For these ranges, the dependency of RDO eigenfrequency to W and β is larger than
 618 for the range of R seen in observations and CMIP. Thus, we just focus on the dependency to
 619 W and β (Fig. 6b). Both W and β will increase the eigenfrequency similarly. The isofrequency
 620 lines can be crudely approximated as diagonal lines with a negative slope close to -1. Actually,
 621 if the isofrequency lines were exactly diagonal lines with a negative slope equal to -1 (cf. slope

622 of the overlaid diagonal line) and monotonically increasing with W and β in that figure 6b,
 623 that would mean that f would be proportional to the sum $W+\beta$. I.e. $f \sim f_0 + S(W+\beta)$, where f_0
 624 and S would be approximately constant within the CMIP parameter range (with approximately
 625 $f_0 \sim 0.008 \text{ month}^{-1}$ and $S \sim 0.09$). The inter-model diversity in RDO eigenfrequency (or also
 626 qualitatively in ENSO spectral shape) should hence be approximately proportional to the sum
 627 $W+\beta$. Indeed, the ENSO spectral shape in CMIP models is roughly linearly related to $W+\beta$, as
 628 evidenced by Fig. 6c, with a high inter-model correlation of 0.75 between $W+\beta$ and ENSO
 629 spectral shape among models. The correlation drops to only 0.43 or 0.30 when only W or β
 630 alone is used instead of their sum $W+\beta$ (note: correlation with W alone here less good than in
 631 Lu et al. (2018) because here W is evaluated from the multi-regression fit for the RDO; in their
 632 study the pure RO is considered, so that their recharge term F_{I_RO} will then mix statistically F_I
 633 and β contributions). The sum $W+\beta$ thus explains well the diversity of ENSO frequency among
 634 CMIP models. It represents the combined effects of the recharge and delayed feedback
 635 processes on ENSO frequency diversity in CMIP models.

636 The eigen growth rate σ_r has a complex dependence to W and β (Fig. 6a; σ_r dependence to
 637 W , β and R separately shown in Suppl. Fig. S6abc). Around CMIP MMM, an increase in W
 638 increases σ_r , i.e. decreases the damping, while an increase in β increases the damping. Around
 639 W and β default values, σ_r increases quasi-linearly with R (Suppl. Fig. S6c), i.e. with the
 640 Bjerknes feedback: $\sigma_r \approx 0.6*(R-0.04)$ (for the RO, we would have $\sigma_r = (R_{RO}-\epsilon)/2 \approx 0.5*R_{RO}$;
 641 Jin et al. 2020). The diversity of these parameters may relate to the diversity in ENSO frequency
 642 and ENSO amplitude among models. Interestingly, obs1 as well as about half of CMIP models
 643 are near criticality (i.e. almost undamped). If the combination of the three parameters leads to
 644 a sufficiently large σ_r so as to become positive, it would lead to an instability. Could this explain
 645 some ENSO non-linearities? Or some tipping point effects? Further work would be needed to
 646 test this.

Dependency of RDO eigenvalues to parameters



647

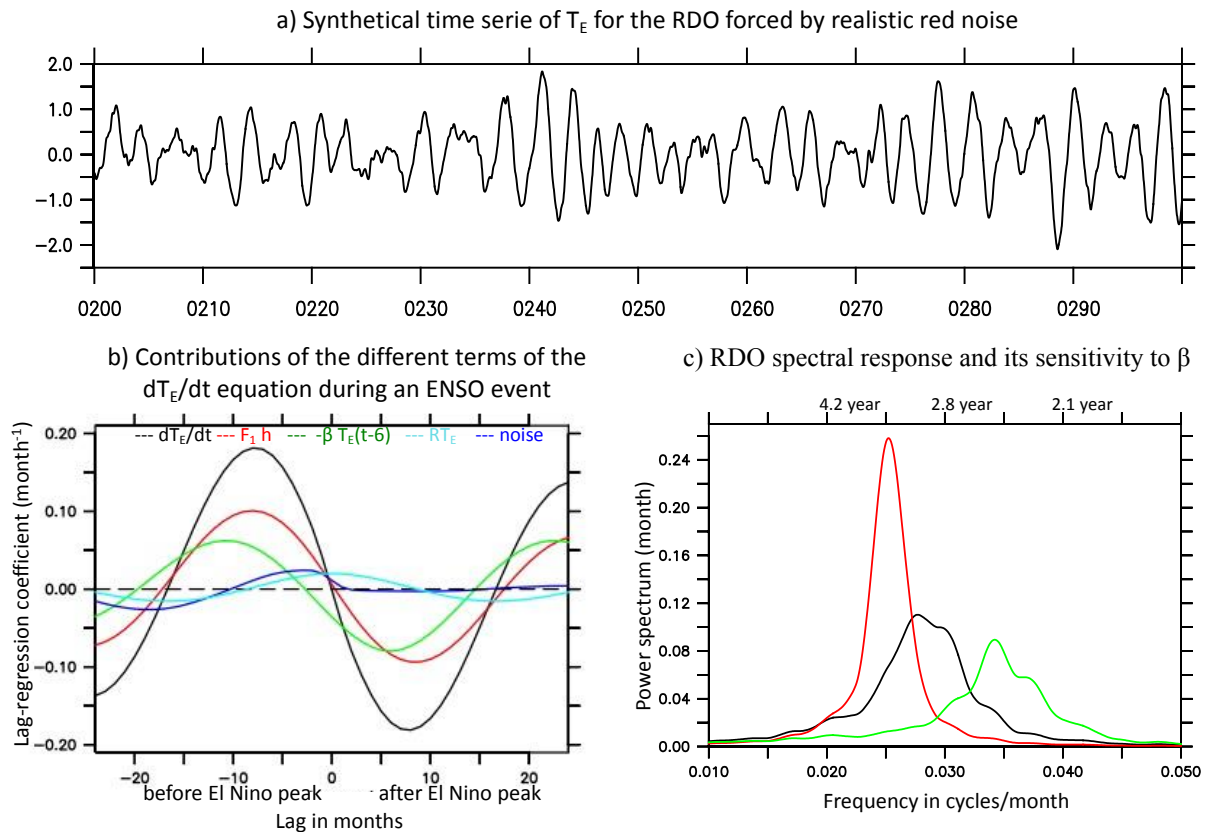
648 **Fig. 6. Dependency of RDO eigenfrequency to parameters. a)** RDO eigen growth rate (color shading;
649 month⁻¹) as a function of both β and Wyrтки angular frequency $W=(F_1F_2)^{1/2}$ (W being the eigen angular
650 frequency of the pure RO). **b)** same as a for eigenfrequency (blue diagonal line showing $W= -\beta + constant$).
651 **c)** ENSO spectral shape metric (higher values when ENSO frequency increases, cf. section 2) as a function
652 of the sum $W+\beta$ for each CMIP5 (blue) and CMIP6 (purple) model (with their MMM in squares). In panels
653 a and b, circles show obs1 (red) and obs2 (green).

654

655 *e. RDO response to stochastic forcing*

656 Now we analyze the RDO response to stochastic forcing. Stochastic forcing is required, as
657 the RDO is damped for the default parameters. To better understand the conceptual model
658 behavior, we have forced the RDO by adding two realistic red noises, one, μ_T , in the right-hand
659 side (rhs) of T_E equation (8) and one, μ_h , in the rhs of h equation (9). The STDs of these red
660 noises ($\sigma_T \approx \sigma_h \approx 0.18$ month⁻¹) are inferred from the two observed residuals of the linear
661 regression fits of equations (8) and (9). These noises are chosen to be independent, as the two
662 observed residuals are only weakly correlated (shared variance < 10%; see also Takahashi et
663 al. 2019 sensitivity tests on this kind of noise choice). Fig. 7a firstly shows an example of the
664 T_E synthetical time series obtained for default parameter values. The system has oscillations,
665 with a frequency modulation, and with low and high-variance decades (similar results over the
666 full 1000-yr period; not shown). The RDO spectral response consistently has a broad peak
667 between ~ 4 and ~ 2 -yr periods, maximal at ~ 3 -yr (black curve in Fig. 7c), roughly consistent
668 with observed ENSO. Fig. 7b shows the typical evolution of the different terms of the dT_E/dt
669 equation during an ENSO event (i.e. lag-regression onto T_E) for the default synthetic time series
670 shown in Fig. 7a. h (red curve) leads T_E (light blue) by ~ 10 months, i.e. about a quarter of the
671 ENSO period, as expected because a recharge in h favors a positive T_E (Equation 8), as for
672 usual RO. The delayed feedback effect has a different timing. It is the first to favor the phase
673 reversal, followed by the recharge process. In the RDO, the *negative* noise forcing at lag ~ -20
674 months can already force indirectly the following El Niño onset, notably through the delayed
675 feedback 6 months later. And a *positive* noise (from about -10 months before ENSO peak) can
676 of course also force an El Niño directly.

677



678
 679 **Fig. 7. RDO response to stochastic forcing.** **a)** Synthetical time series of T_E for the linear RDO forced by
 680 realistic red noises (added to the rhs of equations 8 and 9; $\text{STD}(T_E)=0.7$ for 1000-yr). **b)** The different terms
 681 of the dT_E/dt equation (8) lag-regressed onto T_E , showing their various contributions (unit: month^{-1} per STD
 682 of T_E) during the evolution of an ENSO event: dT_E/dt (black), $F_1 h$ (red), $-\beta T_E(t-\eta)$ (green), RT_E (cyan; which
 683 also indicates ENSO phase), red noise forcing σ_T (dark blue). The x-axis shows the lag, with negative lags
 684 before ENSO peak and positive lags after ENSO peak. **c)** RDO spectral response (i.e. frequency response to
 685 white noise: power spectrum of T_E for a 1000-yr long white noise forcing) for β default value (0.08 month^{-1} , black),
 686 a larger β (0.13 month^{-1} , green; i.e. closer to pure DO with a QuasiBiennial QB mode) and a smaller
 687 β (0.05 month^{-1} , red; i.e. close to pure RO with a QuasiQuadriennial QQ mode; for the small β case, the much
 688 larger power spectrum is divided by 3 for visualization).
 689

690 While most contributions are qualitatively similar in obs1 and obs2 (shown in Supp. Fig.
 691 S7 for comparison) and are within observations confidence intervals, there are also interesting
 692 differences. Observed asymmetries are - by construction - not captured by the simplest RDO
 693 framework with only linear terms (cf. sections 5 and 6), e.g. : 1) non-linearities and external
 694 forcing included in the residual in equations (8) and (9) are possibly large during ENSO
 695 development and mature phase, and damp the reversal to ENSO opposite phase (with the
 696 observed 2nd peak of the residual at ~ 10 -month lag not seen for the synthetic time series); 2)
 697 the advective-reflective delayed negative feedback seems to play a larger and more systematic
 698 role for ENSO events termination and for La Niña onset than for El Niño onset (Suppl. Fig. S7
 699 composites). In summary, within its linear limits, the linear nonseasonal RDO framework

700 qualitatively matches observations and could help us to better understand the interplay between
701 the recharge and advective-reflective delayed feedbacks in shaping the real-world ENSO.

702 To illustrate how sensitive the RDO spectral response is to the strength of the advective-
703 reflective delayed negative feedback β , Fig. 7c shows the RDO spectral response for a smaller
704 (0.05) or larger (0.13) β (its default value being 0.08 month⁻¹). In the small β case, the response
705 is of lower frequency (period~3.3-yr) and less damped, with a sharper peak (closer to RO
706 harmonic oscillator behavior). In the large β case, the response is of higher frequency
707 (period~2.5-yr) and highly damped, with a broader spectral range (closer to DO behavior). This
708 broader spectral response can be explained because the DO has a much broader spectral
709 response, thanks to its infinity of eigenmodes, than the RO which has only one eigenfrequency.
710 I.e. the delayed negative feedback brings spectral diversity to the RDO spectrum. To sum up,
711 ENSO spectrum and amplitude are very sensitive to β , i.e. to the advective-reflective delayed
712 negative feedback strength (itself related to the climatological SST zonal gradient).

713 *f. ENSO spatial-temporal diversity in the RDO framework*

714 The RDO model, even in its simplest form, can also explain part of ENSO spatial diversity.
715 To illustrate this, we can exploit the multivariate linear regression based on equation (8.1) for
716 the tendency of temperature $T(x,y,t)$ anywhere in the tropical Pacific. As seen earlier (Fig. 2ab),
717 the $\beta(x,y)$ contribution reminds us of the central Pacific (CP) El Niño spatial pattern, and the
718 $F_I(x,y)$ contribution reminds us of the eastern Pacific (EP) El Niño pattern. Hence, when the
719 RDO is forced by noise, we will have different spatial patterns of $T(x,y,t)$, closer to CP or EP
720 spatial pattern depending on the relative contributions of β and F_I terms (that notably depend
721 of the frequency of stochastic forcing). The spatial and temporal properties of ENSO are thus
722 strongly dependent on which of the recharge F_I or delayed β effect is locally dominant. Indeed,
723 we can reconstruct “offline” $T(x,y,t)$ at any location by integrating temporally Equation 8.1 (i.e.
724 the linear combination of $T_E(t)$, $h(t)$ and $T_E(t-\delta)$), after having obtained synthetic time series of
725 T_E and h from the RDO forced by noise (e.g. time series shown in Fig. 7a). As an example, we
726 have reconstructed $T(x,y,t)$ from the 1000-yr long synthetic time series of T_E and h , using the
727 averages of obs1 and obs2 for the regression coefficients $R(x,y)$, $F_I(x,y)$ and $\beta(x,y)$
728 (reconstructed $T(x,y,t)$ is high-pass filtered, like long obs2, to remove multidecadal variability
729 that can arise from the time integration, so as to focus on interannual timescales). We can then
730 compare for instance reconstructed Niño4 (CP) and Niño3 (EP) SST variabilities. Niño4 is of
731 significantly higher frequency than Niño3, with an almost-doubled spectral shape metrics: 2.7
732 for Niño4, 1.5 for Niño3. This is because Niño3 is dominated by $F_I h$, and h is essentially the

733 integral of T_E , and thus of lower frequency than T_E . While Niño4 is dominated by $-\beta T_E(t-\eta)$,
734 which has the same spectrum as T_E . The RDO can thus, even in its simplest form, simulates
735 some ENSO spatiotemporal diversity. The RDO captures the fact that part of the CP variability
736 is more biennial (QB) because the delayed feedback is dominant there, and that the EP
737 variability is more quadriennial (QQ; e.g. Wang and Ren 2017) because the slower recharge
738 feedback is dominant there. Note that additionally, there is in observations a decadal
739 component of CP ENSO (Behera and Yamagata 2011, Sullivan et al. 2016, Capotondi et al.
740 2020) that could be implemented in the RDO framework, e.g. through a decadal modulation of
741 the mean state (e.g. Zhang et al. 2019) modulating RDO parameters (possibly through STCs;
742 Zeller et al. 2019, 2021).

743
744

745 **5. Non-linear seasonal RDO**

746

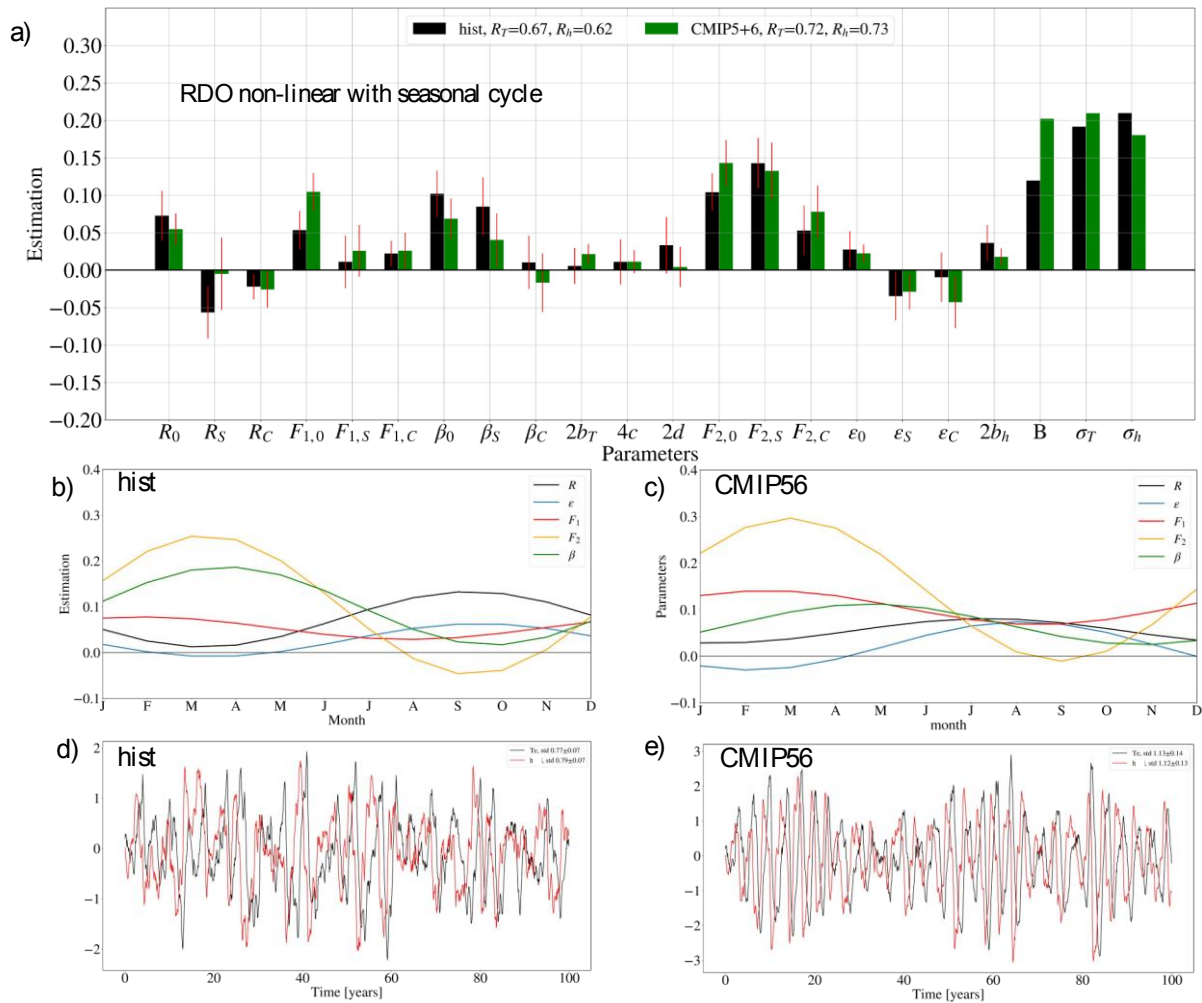
747 The simplest RDO framework, linear and non-seasonal, can be obviously improved by
748 adding: 1) the seasonal cycles of RDO parameters, 2) asymmetries/non-linearities, e.g.
749 quadratic terms and a multiplicative noise. We have added them separately, and then all
750 together in the most comprehensive RDO version, the non-linear seasonal RDO. We have also
751 tested the influence of each non-linear parameter on the phase diagram trajectories
752 (Supplementary Materials). The take-home message is that, with seasonal cycle, non-linearities
753 and multiplicative noise added, the RDO is even more realistic, and still has robust and highly-
754 significant F_1 and β coefficients, i.e. the non-linear seasonal RDO is more realistic than the
755 non-linear seasonal RO or DO.

756 *a. Seasonality of RDO parameters*

757 Knowing the ENSO observed seasonal phase-locking, the first improvement to do to the
758 RDO is to allow all its parameters to vary seasonally. The simplest seasonality we can add for
759 parameters is, for instance for β parameter, of the form $\beta = \beta_0 + \beta_{seas} \sin(\omega t - \lambda) = \beta_0 + \beta_S \sin(\omega t)$
760 $+ \beta_C \cos(\omega t)$. $\beta = \beta_0 + \beta_{seas} \sin(\omega t - \lambda)$ looks mathematically ‘nicer’, by having amplitude β_{seas}
761 and phase λ separately, than $\beta = \beta_0 + \beta_S \sin(\omega t) + \beta_C \cos(\omega t)$. Yet the later form is more
762 convenient, notably to compute intervals of confidence and when we want to compare different
763 observations and/or models, as the phase λ is modulo(2π) and we for instance cannot compute
764 a multi-model mean (MMM) of λ directly from the λ of each model (it would only be possible

765 by first computing the β_S and β_C MMM, to then convert them into β_{seas} and λ values). And β_S
766 and β_C are straightforward to interpret: here $t=0$ on January 1st, so that $\sin(\omega t)$ is maximum on
767 April 1st and minimum on October 1st, and $\cos(\omega t)$ is maximum on January 1st and minimum
768 on July 1st. As the number of parameters to fit is now multiplied by 3, i.e. 9 for T_E tendency
769 equation and 6 for h one, we essentially focus on the results based on our longest reanalysis
770 data, ‘hist’, and on CMIP historical runs, for which the number of effective degrees of freedom
771 is sufficiently large (obs2 still shown in Suppl. Fig. S8; note however the larger uncertainty
772 bars when estimating RDO seasonally-varying parameters with obs2; obs1 is not shown, being
773 obviously too short). In a nutshell, the correlation skills r_T and r_h of dT_E/dt and dh/dt equations
774 strongly increase by including the seasonal cycle (compare panel c to panel b in Suppl. Fig.
775 S8). And the RDO still has a robust and highly-significant β coefficient for the delayed
776 feedback (with β_0 even two times larger than $F_{1,0}$ in hist; Suppl. Fig. S8c).

777 Let us now describe each parameter’s seasonal cycle (Fig. 8abc shown here for the non-
778 linear version of the seasonal RDO, which has a similar seasonality to its linear version; cf.
779 comparison of each parameter’s seasonal coefficients in Suppl. Fig. S8). We here focus on hist-
780 based parameters, as this should be our most realistic estimates, keeping in mind the limit of
781 historical oceanic reanalyses, the actual “truth” being possibly between hist, shorter obs2, and
782 CMIP estimates. To interpret the coefficients seasonality, keep in mind that their actual
783 contribution to T_E and h tendencies will be the product of the coefficient and of its associated
784 term. E.g. $F_2 T_E$ actual contribution to dh/dt will be larger in boreal winter than F_2 alone, when
785 T_E interannual STD peaks. Let us start with R . R has a strong seasonal cycle, with $-R_S \sim R_0 > 0$.
786 R is the largest around September and the lowest in March, favoring a peak of ENSO around
787 December, as expected from previous studies (e.g. Jin et al. 2020). F_1 has a relatively-weak
788 seasonal cycle, being slightly larger around January-March and weaker around July-
789 September, likely because the climatological equatorial upwelling in the central-eastern Pacific
790 (170°W-120°W) is the strongest in January-March and the weakest in July-September. β_0 is
791 highly significant (even above 99% level) and larger than $F_{1,0}$ in hist (but not in obs2 and
792 CMIP). β has a large seasonal cycle, dominated by β_S , i.e. is maximal in spring, possibly
793 because of the seasonal cycle in the product $C\mu_C$ between the coupling parameter μ_C and the
794 climatological zonal SST gradient C (cf. section 3.2), and/or because of Indian Ocean Dipole
795 (IOD, an equivalent of El Niño for the Indian Ocean, e.g. Saji et al. 1999) delayed influence
796 (Izumo et al. 2010) likely partly included in the delayed term. The difference between F_1 and
797 β seasonal cycles further confirms that F_1 and β terms represent distinct physical processes.



798

799

Fig. 8. The non-linear seasonal RDO. a) bar plot showing each parameter of equations 11 and 12, estimated by a multivariate fit (with B and σ_T estimated as in An et al. 2020). To represent the actual contribution of the non-linear terms for strong ENSO events, their parameters are multiplied by 2 for quadratic terms and by 4 for the cubic term, for a fair comparison with linear parameters such as R , F_1 and β (as for a strong 2STD event, we will have e.g. $F_1 T_E = 2F_1$, $b_T T_E^2 = 2*2b_T$, $c T_E^3 = 2*4c$). The correlation skills r_T and r_h are given above the bar plot. For hist (black), the red line shows the 95% interval of confidence. For CMIP MMM (green), it represents the ± 1 inter-model STD (among all CMIP). b) seasonal cycles of the main RDO parameters estimated from the fit on hist. c) same as b but for CMIP MMM. d) 100-yr long timeseries of hist-based nonlinear seasonal RDO (STD given for the average of fifty 100-yr long timeseries). e) as d, but for CMIP-based RDO.

809

810

811

812

813

814

For dh/dt equation (1), which is the same for the RO and RDO, previous studies (e.g. Chen and Jin 2020) of the RO had usually considered F_2 to be seasonally-constant, supposing that its seasonal cycle was negligible. Actually, F_2 has a large significant and robust seasonality in all our estimates (Fig. 8abc and Suppl. Fig. S8). This is the first study to our knowledge that shows this F_2 seasonality for the RO/RDO recharge equation. F_2 represents the efficiency of

815 the recharge for a given T_E anomaly (e.g. 1K). $F_{2,S}$ is as large as the constant component $F_{2,0}$
816 in hist. F_2 is maximum in early spring (Feb.-Apr.) and negligible in early autumn (Aug.-Oct.),
817 with of course differences among observations and CMIP estimates. ε_0 is weakly positive,
818 $\sim 0.02 \text{ month}^{-1}$, with some seasonal cycle, maximum in hist ($\varepsilon_S < 0$) in fall and negligible in
819 spring. Taking into account all these seasonal cycles lead to a more realistic RDO behavior
820 with seasonal phase-locking and richer combination tones (e.g. Stuecker et al. 2013), especially
821 when including non-linearities, i.e. the full non-linear seasonal RDO version described
822 hereafter.

823 *b. Adding non-linearity: the non-linear seasonal RDO*

824 Above we had neglected asymmetries/non-linearities for the sake of simplicity. The next
825 step is to include such terms. We can add to the dT_E/dt equation (8) the quadratic and cubic
826 terms $+b_T T_E^2 - c T_E^3 + d T_E h$. They notably represent the non-linear response of convection
827 (and of related windstress) to T_E (e.g. Choi et al. 2013; Takahashi et al. 2019; Jin et al. 2020;
828 An et al. 2020; Dommenget and Al Ansari 2022; Srinivas et al. 2022, 2024). We can also add
829 a multiplicative noise (e.g. Jin et al. 2007; Graham et al. 2015, their equation 23). We can add
830 to the dh/dt equation (9) a quadratic term $-b_h T_E^2$. I.e. the discharge during an El Niño of
831 amplitude T_{E_0} is larger than the recharge during a La Niña of similar amplitude $-T_{E_0}$, notably
832 because equatorial zonal windstress anomalies are of larger amplitude and fetch (Srinivas et al.
833 2024) and are further to the east (e.g. Izumo et al. 2019; note that we could add even further
834 complexity, e.g. a non-linearity of the delayed term, as in DiNezio and Deser 2014, and a state-
835 dependency of F_2 as in Iwakiri and Watanabe 2022 to even better simulate long-lasting La
836 Niña). Thus, the non-linear seasonal RDO equations are:

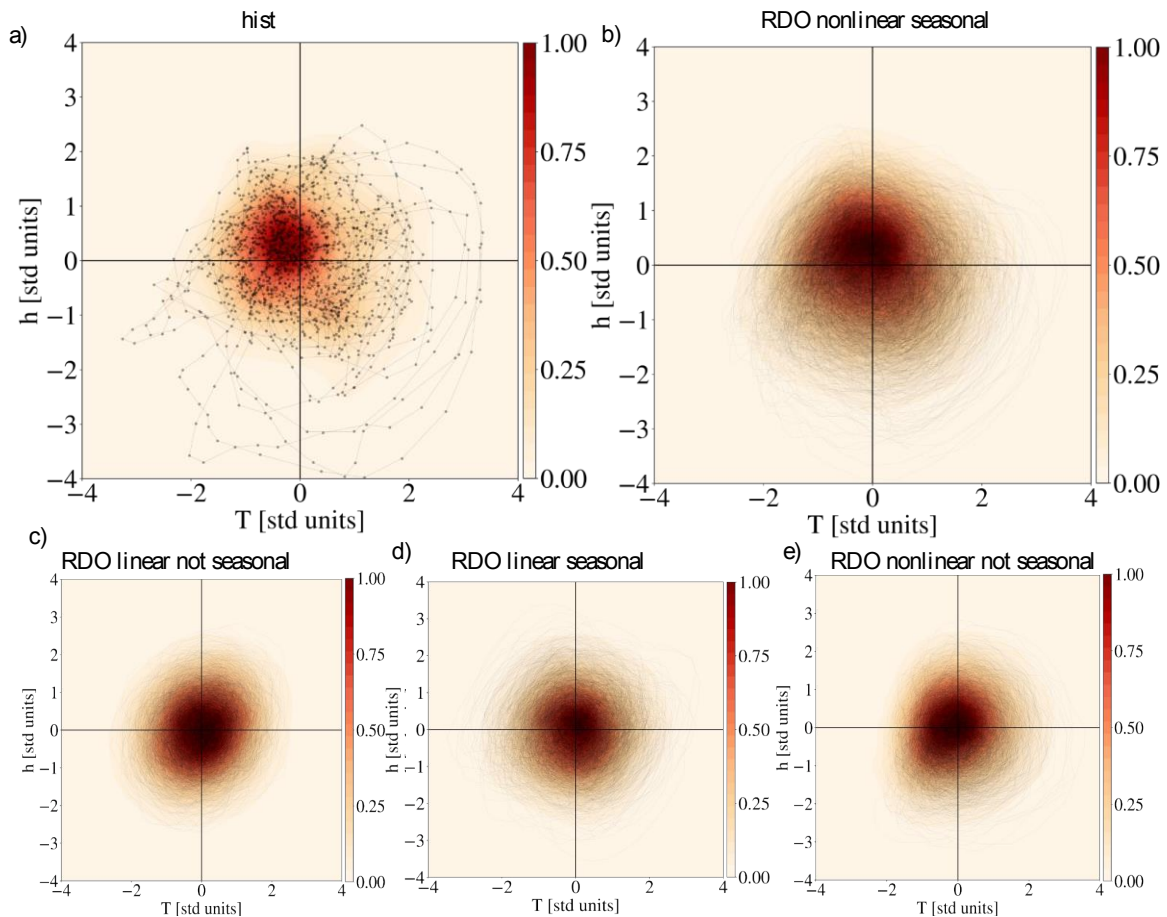
837
838
$$dT_E/dt = R T_E + F_1 h - \beta T_E(t-\eta) + b_T T_E^2 - c T_E^3 + d T_E h + \mu_T(1+B T_E) \quad (11)$$

839
$$dh/dt = -F_2 T_E - \varepsilon h - b_h T_E^2 + \mu_h \quad (12)$$

840
841 Where μ_T and μ_h are red noises with standard deviations σ_T and σ_h respectively, and with
842 a 5-day decorrelation time (cf. section 2). B represents the strength of the multiplicative noise
843 (i.e. larger noise during El Niño than during La Niña). Here we have tried to keep the notations
844 as much as possible consistent with the (soon open-source) Community RO model under
845 development presently, to which we participate (Vialard et al. 2023, personal communication;
846 see also review of An et al. 2020).

847 Now we have 12 parameters to fit for equation (11) (as we assume no seasonality of the
848 non-linear terms, and as B and σ_T are separately estimated from the residual distribution), and
849 7 parameters for equation (12), shown in Fig. 8a and Suppl. Fig. S8. The first result is that the
850 RDO is still robust and better than the RO, when including non-linearity and multiplicative
851 noise in addition to seasonality. β_0 is large and significant (even at the 99% level in hist and
852 CMIP). Including this delayed effect to the RO clearly increases the skill and explained
853 variance of dT_E/dt equation (in hist: correlation skill $r_T=0.60$ to 0.67 , i.e. a relative increase of
854 explained variance by 25%, r_T^2 increasing from 36% to 45%; in CMIP: $r_T=0.67$ to 0.72 , i.e.
855 increase of explained variance by 16%, r_T^2 increasing from 45% to 52%). The results for CMIP
856 and hist are qualitatively consistent, but quantitatively different. The largest differences are for
857 F_I and β relative contributions, β being larger than F_I in hist (vice versa for CMIP), as
858 aforementioned. The nonlinearities d and b_h are stronger in hist. Fig. 8de shows the generated
859 synthetic time series of T_E , for hist and CMIP-MMM parameters respectively. They are quite
860 different too. The synthetic time series for hist look more realistic (asymmetry, irregularity).
861 Most CMIP models have a bias in ENSO nonlinearity/asymmetry (Hayashi et al. 2020); thus,
862 that their RDO also fails to reproduce asymmetry is expected. The larger irregularity of hist-
863 based RDO could be partly explain by the larger β/F_I ratio in hist than in CMIP. Fig. 9b shows
864 the trajectories in the (T_E, h) phase space of the hist-based nonlinear seasonal RDO. They are
865 qualitatively similar to hist ones (Fig. 9a), with similar asymmetries, namely larger extreme El
866 Niño events with larger and faster discharges than extreme La Niña (see e.g. Iwakiri and
867 Watanabe 2022 for processes driving long-lasting La Niña).

868



869

870 **Fig. 9. a) Phase diagram (T_E, h) of hist observations/reanalysis.** The trajectories are shown in thin grey
 871 curves, and their probability density function (PDF) in color shading. **b) phase diagram ($T_{E_synthetic}, h_{synthetic}$)**
 872 of 50 100-yr long synthetic time series obtained by forcing hist-based non-linear seasonal RDO by red
 873 noises. **c) as b but for the hist-based simplest linear RDO without seasonality (and without multiplicative**
 874 **noise, i.e. $B=0$).** **d) as c but adding seasonality.** **e) as c but adding non-linearities (including multiplicative**
 875 **noise).** Note that, when the distribution is shifted with extreme El Niño and discharge being stronger, the
 876 PDF maximum, i.e. the most likely points, is shifted towards slightly negative T_E and positive h anomalies
 877 (mean removed).

878

879 To highlight visually the role of each non-linear term in the non-linear seasonal RDO (for
 880 the set of parameters obtained from hist), Suppl. Fig. S9 shows RDO phase diagram when each
 881 of them is artificially removed. E.g. the absence of $b_h T_E^2$ in S9b leads to less extreme discharge
 882 (there is still some asymmetry in the recharge/discharge due to the non-linear terms in the
 883 dT_E/dt equation, but weaker than with b_h included). I.e. S9b shows that $b_h T_E^2$ role makes the
 884 discharge more extreme during strong El Niño. Multiplicative noise associated to B sustains
 885 strong El Niño (i.e. the lower-right quarter of the phase diagram; Suppl. Fig. S9d). The role of
 886 $b_T T_E^2$ should be to increase El Niño and reduce La Niña. However, in the hist case shown here

887 where b_T is weak, this effect is hard to see in Suppl. Fig. S9e (it would be more seen in CMIP-
888 based RDO case). The cubic term $-cT_E^3$ reduces very extreme T_E events (for both El Niño and
889 La Niña events; Suppl. Fig. S9f). The $d T_E h$ favors larger « La Niña+discharged state » events
890 in the lower-left quadrant and diminishes them in the upper-right quadrant, ‘tilting’ the phase
891 diagram. Note that this detailed analysis of the role of each non-linear term remains
892 qualitatively valid for the RO framework.

893

894 **6. Conclusion**

895 *a. Summary*

896 We have developed a hybrid recharge-delayed oscillator, the RDO, more realistic than the
897 stand-alone RO or DO (schematics in Fig. 1). This RDO can help us to improve our theoretical
898 understanding of ENSO and of its irregularity, diversity and complexity. We have shown that
899 the ENSO temperature tendency dT_E/dt results essentially from the combination of two distinct
900 negative feedback mechanisms with distinct spatial distributions, in addition to the Bjerknes
901 positive feedback:

902 (1) the slow long-term recharge during La Niña (discharge during El Niño) with a
903 timescale of $\sim 1-2$ years. This slow recharge process takes place over the equatorial and
904 southwestern Pacific independently of the fast tilt mode, i.e. “ h ” = h_{ind_eq+sw} . This long-term
905 recharge firstly affects the oceanic mixed layer temperature tendency in the central-eastern
906 equatorial Pacific by favoring advection of warmer subsurface waters by the climatological
907 STCs/TCs and EUC towards the equatorial upwelling, i.e. $-U_{clim} \partial T'/\partial x$ and $-W_{clim} \partial T'/\partial z$.

908 (2) the advective-reflective delayed negative feedback of ~ 6 months. This delayed
909 feedback firstly affects the oceanic mixed layer temperature tendency in the western-central
910 equatorial Pacific by anomalous upper layer zonal current advecting the climatological
911 temperature zonal gradient through wave processes, i.e. $-U' \partial T_{clim}/\partial x$.

912 These two *distinct* processes are both essential for equatorial Pacific SST variability and
913 so for the coupled system. Hence, rather than stating that the recharge oscillator (RO) and
914 delayed oscillator (DO) are two different formal ways of representing the same physical
915 process as done usually in previous studies, the present results based on observations and 79
916 CMIP models show that these two different physical processes with different timescales should
917 be distinguished in the equations. They can be formally incorporated in the RDO. This hybrid
918 oscillator has qualitatively-realistic spectral characteristics (with a wider spectral peak than the

919 RO) and lead-lag relationships. The inclusion of the delayed term $-\beta T_E(t-\eta)$ [with $\eta = 6$ months]
920 favors a more irregular, and possibly chaotic, behavior when forced by stochastic forcing.
921 RDO eigenvalues are highly sensitive to both the Wyrтки angular frequency $W=(F_1F_2)^{1/2}$ and
922 the β parameter (representing respectively the strengths of the RO and DO components),
923 themselves sensitive to the mean state (and thus to CMIP model biases). We have notably
924 shown that the main RDO eigenfrequency is approximately linearly related to the sum $W+\beta$.
925 I.e. ENSO frequency increases not only if W is larger, but also if the advective-reflective
926 delayed feedback is larger. Furthermore, by taking into account that the advective-reflective
927 delayed feedback (respectively recharge feedback) is larger in the western-central Pacific
928 (respectively eastern equatorial Pacific), each feedback having its own time scale, we can
929 reconstruct equatorial SST at any longitude within the RDO framework, and simulate some
930 ENSO spatiotemporal diversity.

931 Finally, we have shown that the simple RDO framework is robust and more realistic than
932 the RO framework, and could be significantly improved by adding: 1) the seasonal cycles of
933 RDO parameters, 2) asymmetries/non-linearities, e.g. quadratic/cubic terms and a
934 multiplicative noise. These non-linearities added to seasonality further increase the system's
935 complexity and possibly make it more irregular and chaotic, and thus increase potentially
936 ENSO spatiotemporal diversity. We have also investigated the influence of each non-linear
937 term on the (T_E, h) phase space trajectories, an investigation that is also useful for the RO
938 framework.

939 *b. Discussion on possible improvements of the RDO*

940 This latest and more refined RDO version could still be further improved by adding: 1)
941 influences external to the tropical Pacific, 2) a third box in the model to explicitly allow for
942 more spatial diversity of ENSO events.

943 External influences to add are notably the two-way interaction of the IOD with ENSO (e.g.
944 Izumo et al. 2010, Luo et al. 2010, Jourdain et al. 2016), the mean tropical Indian and Atlantic
945 Oceans warming/cooling, which act as negative feedbacks to ENSO (e.g. Dommenges and Yu
946 2017), and the north and south tropical Pacific (e.g. Alexander et al. 2010). E.g. the IOD can
947 also force the advective-reflective delayed feedback: a negative IOD forces easterly anomalies
948 in the western Pacific, thereby favoring positive zonal current anomalies ~ 6 months later in the
949 western-central equatorial Pacific. Hence a perspective would be to add a third variable in the
950 system, the IOD as a forcing external to the tropical Pacific coupled to ENSO (mathematically

951 like in Kug and Kang 2006 and Frauen and Dommengeset 2012, but for the IOD rather than for
952 the Indian Ocean basin-wide SST).

953 Here our original aim was not to capture the various ENSO flavors, i.e. the ENSO diversity
954 continuum from extreme EP El Niño to CP El Niño Modoki events, that have partly distinct
955 global teleconnections. For the sake of simplicity, we have focused on ENSO events defined
956 with the usual Niño3.4 region. Still, the RDO framework has allowed us to reconstruct ENSO
957 spatiotemporal diversity, at least partly, even without adding non-linear terms (Fig. 2ab). While
958 in the RO framework, nonlinearities are needed to capture ENSO diversity (Thual and Dewitte
959 2023). Adding seasonality and nonlinearities to the RDO (section 5) can increase simulated
960 ENSO diversity/complexity even more. The next step is to implement our RDO approach in a
961 three-box conceptual model (i.e. west, central and east Pacific boxes), such as in Fang and Mu
962 (2018), Geng et al. (2020) and Chen et al. (2022) but with a physically-based formalization of
963 the equations inspired by the above RDO approach.

964 *c. Implications and perspectives*

965 The RO still remains useful, being the simplest ENSO conceptual model. We still
966 appreciate its use. But it implicitly mixes two physically different processes, the zonal
967 advective and thermocline feedbacks. The RDO does not. It is more physical and captures the
968 spatial and frequential diversity of ENSO, while still remaining simple enough.

969 The RDO set of equations may formally look partly like a simplification of the unified
970 oscillator of Wang (2001), which was shown by Graham et al. (2015) to be less realistic than
971 the simple DO. However, here the RDO set of equations is for (T_E, h_{ind_eq+sw}) orthogonal space
972 instead of (T_E, h_{eq}) non-orthogonal space. And the various terms of the RDO equations represent
973 clear distinct physical mechanisms, conversely to the unified oscillator as pointed out by
974 Graham et al. (2015).

975 For ENSO operational forecast diagnostics, Izumo and Colin (2022) have shown that the
976 pair of coordinates (T_E, h_{ind_eq+sw}) is more relevant to describe the RO system trajectory than the
977 usual pairs (T_E, h_{eq}) or (T_E, h_w) . Here the RDO realism furthermore suggests that an additional
978 useful term to take into account to operationally diagnose the system state is $T_E(t-\eta)$ as an
979 indicator of the advective-reflective delayed negative feedback effect, e.g. when diagnosing
980 the present oceanic state or for operational forecasts. The RDO framework should also be
981 useful to study ENSO predictability with information theory (Fang and Chen 2023).

982 Background interdecadal changes in the sum $W+\beta$ may favor the Quasi-Quadriennial (QQ)
983 or Quasi-Biennial (QB) ENSO regimes (e.g. Jin et al. 2020), and might have a role in ENSO

984 ‘regime shifts’ and ENSO diversity. The QQ regime is more associated with large EP El Niño
985 events, for which the thermocline feedback (W) plays a central role. While the QB regime is
986 more associated with moderate CP El Niño events more driven by the zonal advection feedback
987 (β). The RDO framework (possibly adding a third box as suggested above) could help us in
988 understanding the differences between the decades with weak and strong ENSO variances and
989 with differences in ENSO frequency and flavors, possibly related to decadal mean state
990 changes influencing β and W relative strengths (Chen et al. 2022, Chen and Fang 2023), as well
991 as the ENSO response to climate change (e.g. Cai et al. 2021; Shin et al. 2022).

992 There are several pathways to better understand theoretically the RDO model. How
993 combining the RO and DO influences the chaotic behavior of the delayed differential equation
994 system (e.g. Tziperman et al. 1995, Keane et al. 2016), as for the case of two coupled oscillators
995 interacting with noise. A mathematical approach with series as in Power (2011) for the DO
996 could be developed for the RDO. For the nonlinear seasonal RDO, an approach based on a
997 Fokker-Planck equation could help (An et al. 2021). Then we could use an intermediate
998 approach similar to Yu et al. (2015), who combined the RDO to a slab ocean coupled to an
999 AGCM to capture ENSO dynamics and diversity. But we could modify their intermediate
1000 model by (1) using an independent recharge index such as $hind_{eq+sw}$, (2) adding the delayed
1001 negative feedback with its shorter timescale.

1002 The RDO could help us to reconcile ENSO theories. It would be very interesting to redo
1003 detailed analyses testing each ENSO oscillator, such as Graham et al. (2015) approach, and/or
1004 Linz et al. (2014) approach based on transfer functions, in light of the present results, and see
1005 whether theories, climate models and observations would better agree if one keeps RO and DO
1006 processes distinguished through the hybrid RDO framework developed here. To conclude,
1007 using this simple RDO framework could help us to improve ENSO theories, climate model
1008 diagnostics and forecasts.

1009

1010 *Acknowledgments.*

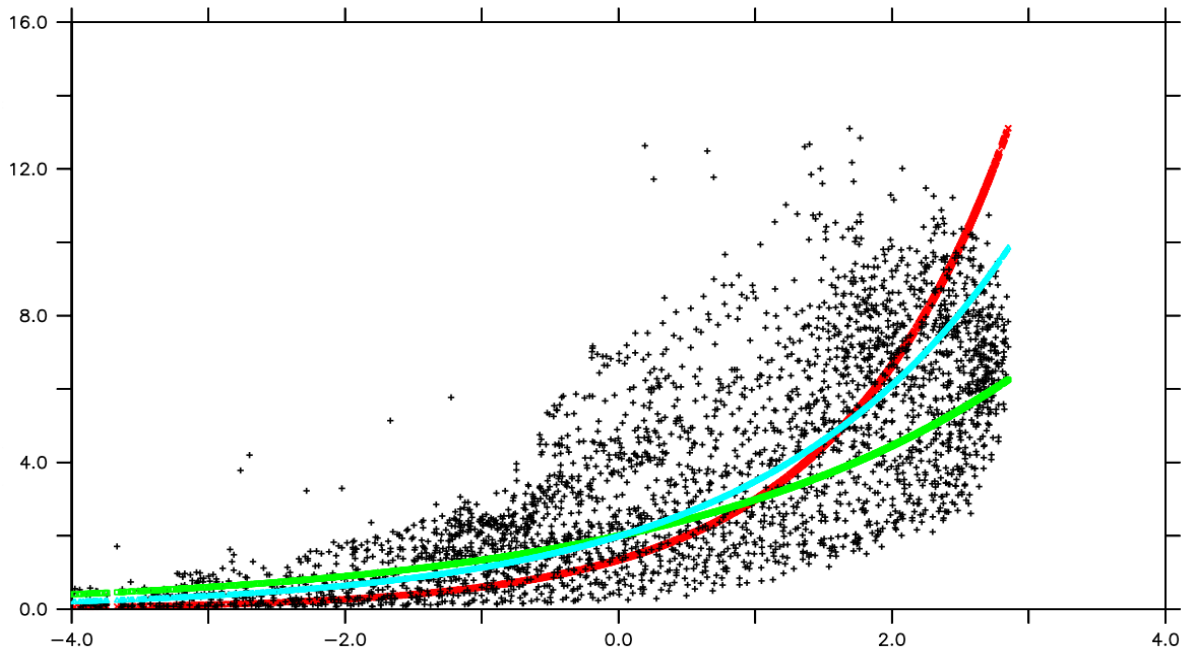
1011 First we would like to thank the three reviewers for their highly-relevant reviews. They
1012 have pushed us to demonstrate even further the RDO framework potential, and that have also
1013 clearly improved the readability of the manuscript. T.I. would like to thank his excellent former
1014 student, Timothée Jullien de Pommerol, for downloading the CMIP6 data, and for
1015 implementing a first Python version of the nonlinear seasonal RO, that B.P. has then further
1016 improved, as well as Jérôme Vialard and Matthieu Lengaigne for interesting discussions at a

1017 much earlier stage of this work. M.C. would like to thank Steven Sherwood for his essential
1018 support, supervision, and mentoring, in particular throughout the pandemic, which made this
1019 collaboration possible. We also would like to thank Soong-Ki Kim, Sen Zhao and Jérôme
1020 Vialard for interactions and discussions on the non-linear seasonal RO implemented by B.P. in
1021 Python in the Community RO Model framework initiated by Jérôme Vialard (who we really
1022 thank for this excellent initiative). T.I. and M.C. would like to thank UPF team for their
1023 continuing support, as well as for hosting M.C. because of COVID-related closed Australian
1024 borders; this latter fact was paradoxically essential for allowing us to make this work mature.
1025 T.I. and B.P. are funded mainly by IRD core budget, with some funding from UMR241
1026 SECOPOL (ex-EIO) laboratory and ANR ARISE project.

1027

1028 *Data Availability Statement.*

1029 All data used is open data: OISSTv2
1030 (<https://psl.noaa.gov/data/gridded/data.noaa.oisst.v2.html>), ERA5 and ORAS5
1031 (<https://www.ecmwf.int>), Copernicus SLA (<https://doi.org/10.48670/moi-00148>), SODA
1032 (<https://www2.atmos.umd.edu/~ocean/>), Tropflux (<https://incois.gov.in/tropflux/>),
1033 HadiSSTv1.1 (<https://www.metoffice.gov.uk/hadobs/hadisst/>), CMIP models (<https://wcrp-cmip.org>), GPCP (<https://psl.noaa.gov/data/gridded/data.gpcp.html>), 20CR
1034 (https://psl.noaa.gov/data/20thC_Rean/). We acknowledge use of NOAA pyferret open
1035 source software for analyses and figures, as well as python.
1036

1038
1039

1040 **Fig. A1. Verification of the approximated formula for the sensitivity of precipitation to RSST.**
 1041 Scatterplot of $Precip_{clim}$ (monthly GPCP precipitation, mm/day) to $RSST_{clim}$ (monthly OISST, regridded on
 1042 GPCP grid; K) in the equatorial Pacific ($5^{\circ}N$ - $5^{\circ}S$, $120^{\circ}E$ - $80^{\circ}W$). The default fit in blue is $P(RSST) = 2 e^{0.56RSST}$,
 1043 the factor 0.56 inspired by Tippett et al. cyclogenesis index. Tests using other exponential forms,
 1044 e.g. $P(RSST) = 2 e^{0.4RSST}$ (green) or $P(RSST) = 2 e^{0.8(RSST-0.5)}$ (red) show that the default approximation is
 1045 qualitatively reasonable. Fig. 2cd redone with these other forms lead to qualitatively-similar results, with
 1046 delayed term contribution increasing if the factor in the exponential increases, as it gives more weight to
 1047 regions with higher climatological RSST (vice versa if it decreases).

1048

REFERENCES

- 1049 Adler, R.F., G.J. Huffman, A. Chang, R. Ferraro, P. Xie, J. Janowiak, B. Rudolf, U. Schneider,
1050 S. Curtis, D. Bolvin, A. Gruber, J. Susskind, and P. Arkin, 2003: The Version 2 Global
1051 Precipitation Climatology Project (GPCP) Monthly Precipitation Analysis (1979-
1052 Present). *J. Hydrometeor.*, 4,1147-1167.
- 1053 Alexander, M. A., Vimont, D. J., Chang, P., & Scott, J. D. (2010). The impact of extratropical
1054 atmospheric variability on ENSO: Testing the seasonal footprinting mechanism using
1055 coupled model experiments. *Journal of Climate*, 23(11), 2885-2901.
- 1056 An, S. I., Tziperman, E., Okumura, Y. M., & Li, T. (2020). ENSO irregularity and
1057 asymmetry. *El Niño Southern Oscillation in a changing climate*, 153-172.
- 1058 An, S. I., Kim, S. K., & Timmermann, A. (2020). Fokker–Planck dynamics of the El Niño-
1059 Southern Oscillation. *Scientific reports*, 10(1), 16282.
- 1060 Ashok, K., Behera, S. K., Rao, S. A., Weng, H., & Yamagata, T. (2007). El Niño Modoki and
1061 its possible teleconnection. *Journal of Geophysical Research: Oceans*, 112(C11).
- 1062 Ballester, J., Bordoni, S., Petrova, D., & Rodó, X. (2015). On the dynamical mechanisms
1063 explaining the western Pacific subsurface temperature buildup leading to ENSO
1064 events. *Geophysical Research Letters*, 42(8), 2961-2967.
- 1065 Ballester, J., Bordoni, S., Petrova, D., & Rodó, X. (2016). Heat advection processes leading to
1066 El Niño events as depicted by an ensemble of ocean assimilation products. *Journal of*
1067 *Geophysical Research: Oceans*, 121(6), 3710-3729.
- 1068 Barnston, A. G., Tippett, M. K., L'Heureux, M. L., Li, S., & DeWitt, D. G. (2012). Skill of
1069 real-time seasonal ENSO model predictions during 2002–11: Is our capability
1070 increasing?. *Bulletin of the American Meteorological Society*, 93(5), 631-651.
- 1071 Barnston, A. G., Tippett, M. K., Ranganathan, M., & L'Heureux, M. L. (2019). Deterministic
1072 skill of ENSO predictions from the North American Multimodel Ensemble. *Climate*
1073 *Dynamics*, 53(12), 7215-7234.
- 1074 Battisti, D. S., & Hirst, A. C. (1989). Interannual variability in a tropical atmosphere–ocean
1075 model: Influence of the basic state, ocean geometry and nonlinearity. *Journal of*
1076 *Atmospheric Sciences*, 46(12), 1687-1712.
- 1077 Bayr, T., Latif, M., Dommenges, D., Wengel, C., Harlaß, J., & Park, W. (2018). Mean-state
1078 dependence of ENSO atmospheric feedbacks in climate models. *Climate*
1079 *Dynamics*, 50(9), 3171-3194.

- 1080 Bayr, T., Wengel, C., Latif, M., Dommenges, D., Lübbecke, J., & Park, W. (2019a). Error
1081 compensation of ENSO atmospheric feedbacks in climate models and its influence on
1082 simulated ENSO dynamics. *Climate dynamics*, 53(1), 155-172.
- 1083 Behera S, Yamagata T. Imprint of the El Niño Modoki on decadal sea level changes.
1084 Geophysical Research Letters. 2010 Dec;37(23).
- 1085 Bellenger, H., Guilyardi, É., Leloup, J., Lengaigne, M., & Vialard, J. (2014). ENSO
1086 representation in climate models: From CMIP3 to CMIP5. *Climate Dynamics*, 42(7),
1087 1999-2018.
- 1088 Blanke B, Raynaud S. Kinematics of the Pacific equatorial undercurrent: An Eulerian and
1089 Lagrangian approach from GCM results. *Journal of Physical Oceanography*. 1997 Jun
1090 1;27(6):1038-53.
- 1091 Bosc, C., & Delcroix, T. (2008). Observed equatorial Rossby waves and ENSO- related warm
1092 water volume changes in the equatorial Pacific Ocean. *Journal of Geophysical Research:*
1093 *Oceans*, 113(C6).
- 1094 Boulanger, J. P., & Menkes, C. (2001). The Trident Pacific model. Part 2: role of long
1095 equatorial wave reflection on sea surface temperature anomalies during the 1993–1998
1096 TOPEX/POSEIDON period. *Climate Dynamics*, 17(2), 175-186.
- 1097 Boulanger, J. P., Menkès, C., & Lengaigne, M. (2004). Role of high-and low-frequency winds
1098 and wave reflection in the onset, growth and termination of the 1997–1998 El
1099 Niño. *Climate dynamics*, 22(2-3), 267-280.
- 1100 Bretherton, C. S., Widmann, M., Dymnikov, V. P., Wallace, J. M., & Bladé, I. (1999). The
1101 effective number of spatial degrees of freedom of a time-varying field. *Journal of*
1102 *climate*, 12(7), 1990-2009.
- 1103 Bunge, L., & Clarke, A. J. (2009). A verified estimation of the El Niño index Niño-3.4 since
1104 1877. *Journal of Climate*, 22(14), 3979-3992.
- 1105 Burgers, G., F.-F. Jin, and G. J. van Oldenborgh (2005), The simplest ENSO recharge
1106 oscillator, *Geophys. Res. Lett.*, 32, L13706, doi:10.1029/2005GL022951.
- 1107 Cai, W., Santoso, A., Collins, M., Dewitte, B., Karamperidou, C., Kug, J. S., ... & Zhong, W.
1108 (2021). Changing El Niño–Southern oscillation in a warming climate. *Nature Reviews*
1109 *Earth & Environment*, 2(9), 628-644.
- 1110 Capotondi, A. (2013). ENSO Diversity in the NCAR CCSM4 Climate Model. *J. Geophys. Res.*
1111 *Oceans* 118, 4755–4770. doi:10.1002/jgrc.20335

- 1112 Capotondi, A., Wittenberg, A. T., Newman, M., Di Lorenzo, E., Yu, J. Y., Braconnot, P., ... &
1113 Yeh, S. W. (2015). Understanding ENSO diversity. *Bulletin of the American*
1114 *Meteorological Society*, 96(6), 921-938.
- 1115 Chen, N., Fang, X., & Yu, J. Y. (2022). A multiscale model for El Niño complexity. *npj*
1116 *Climate and Atmospheric Science*, 5(1), 16.
- 1117 Chen, N., & Fang, X. (2023). A simple multiscale intermediate coupled stochastic model for El
1118 Niño diversity and complexity. *Journal of Advances in Modeling Earth Systems*, 15(4),
1119 e2022MS003469.
- 1120 Chen HC, Jin FF. Fundamental behavior of ENSO phase locking. *Journal of Climate*. 2020
1121 Mar 1;33(5):1953-68.
- 1122 Choi, K.-Y., Vecchi, G. A., & Wittenberg, A. T. (2013). ENSO transition, duration, and
1123 amplitude asymmetries: Role of the nonlinear wind stress coupling in a conceptual
1124 model. *Journal of Climate*, 26(23), 9462–9476. [https://doi.org/10.1175/JCLI-D-13-](https://doi.org/10.1175/JCLI-D-13-00045.1)
1125 00045.1
- 1126 Clarke, A. J. (1994). Why are surface equatorial ENSO winds anomalously westerly under
1127 anomalous large-scale convection?. *Journal of climate*, 7(10), 1623-1627.
- 1128 Clarke, A. J., Van Gorder, S., & Colantuono, G. (2007). Wind stress curl and ENSO
1129 discharge/recharge in the equatorial Pacific. *Journal of Physical Oceanography*, 37(4),
1130 1077-1091.
- 1131 Clarke, A. J. (2008). *An introduction to the dynamics of El Niño and the Southern Oscillation*.
1132 Elsevier.
- 1133 Clarke, A. J. (2010). Analytical theory for the quasi-steady and low-frequency equatorial ocean
1134 response to wind forcing: The “tilt” and “warm water volume” modes. *Journal of*
1135 *physical oceanography*, 40(1), 121-137.
- 1136 Crespo, L. R., M. Belén Rodríguez-Fonseca, I. Polo, N. Keenlyside, and D. Dommenges, 2022:
1137 Multidecadal variability of ENSO in a recharge oscillator framework. *Environ. Res.*
1138 *Let.*, 17, 074008, <https://doi.org/10.1088/1748-9326/ac72a3>.
- 1139 DiNezio PN, Deser C. Nonlinear controls on the persistence of La Niña. *Journal of Climate*.
1140 2014 Oct 1;27(19):7335-55.
- 1141 Dommenges, D., & Yu, Y. (2017). The effects of remote SST forcings on ENSO dynamics,
1142 variability and diversity. *Climate Dynamics*, 49(7), 2605-2624.
- 1143 Fang, X., & Chen, N. (2023). Quantifying the predictability of ENSO complexity using a
1144 statistically accurate multiscale stochastic model and information theory. *Journal of*
1145 *Climate*, 36(8), 2681-2702.

- 1146 Fang, X. H., & Mu, M. (2018). A three-region conceptual model for central Pacific El Niño
1147 including zonal advective feedback. *Journal of Climate*, 31(13), 4965-4979.
- 1148 Fang, X. H., & Zheng, F. (2021). Effect of the air–sea coupled system change on the ENSO
1149 evolution from boreal spring. *Climate Dynamics*, 57(1-2), 109-120.
- 1150 Fedorov, A. V. (2010). Ocean response to wind variations, warm water volume, and simple
1151 models of ENSO in the low-frequency approximation. *Journal of climate*, 23(14), 3855-
1152 3873.
- 1153 Frauen, C., and D. Dommenges (2012), Influences of the tropical Indian and Atlantic Oceans
1154 on the predictability of ENSO, *Geophys. Res. Lett.*, 39, L02706, doi:10.1029/
1155 2011GL050520.
- 1156 Gadgil, S., Joseph, P. V., & Joshi, N. V. (1984). Ocean–atmosphere coupling over monsoon
1157 regions. *Nature*, 312(5990), 141-143.
- 1158 Geng, T., Cai, W., & Wu, L. (2020). Two types of ENSO varying in tandem facilitated by
1159 nonlinear atmospheric convection. *Geophysical Research Letters*, 47, e2020GL088784.
1160 <https://doi.org/10.1029/2020GL088784>
- 1161 Graham, F. S., Brown, J. N., Wittenberg, A. T., & Holbrook, N. J. (2015). Reassessing
1162 conceptual models of ENSO. *Journal of Climate*, 28(23), 9121-9142.
- 1163 Graham, F. S., Wittenberg, A. T., Brown, J. N., Marsland, S. J., & Holbrook, N. J. (2017).
1164 Understanding the double peaked El Niño in coupled GCMs. *Climate Dynamics*, 48(5-6),
1165 2045-2063.
- 1166 Hasegawa, T., T. Horii, and K. Hanawa (2006), Two different features of discharge of
1167 equatorial upper ocean heat content related to El Niño events, *Geophys. Res. Lett.*, 33,
1168 L02609, doi:10.1029/2005GL024832.
- 1169 Hayashi, M., F.-F. Jin, and M. F. Stuecker, 2020: Dynamics for El Niño-La Niña asymmetry
1170 constrain equatorial-Pacific warming pattern. *Nat Commun*, 11, 4230,
1171 <https://doi.org/10.1038/s41467-020-17983-y>.
- 1172 Iwakiri, T., and M. Watanabe, 2022: Multiyear ENSO Dynamics as Revealed in Observations,
1173 Climate Model Simulations, and the Linear Recharge Oscillator. *Journal of Climate*, 35,
1174 4025-4042, <https://doi.org/10.1175/JCLI-D-22-0108.1>.
- 1175 Izumo T (2003) The equatorial undercurrent and associated mass and heat exchanges in the
1176 tropical Pacific: variability, links with El Niño–La Niña events. Ph.D. manuscript,
1177 available at <http://tel.ccsd.cnrs.fr>

- 1178 Izumo, T. (2005). The equatorial undercurrent, meridional overturning circulation, and their
1179 roles in mass and heat exchanges during El Niño events in the tropical Pacific
1180 Ocean. *Ocean Dynamics*, 55(2), 110-123.
- 1181 Izumo, T., Vialard, J., Lengaigne, M., de Boyer Montegut, C., Behera, S. K., Luo, J. J., ... &
1182 Yamagata, T. (2010). Influence of the state of the Indian Ocean Dipole on the following
1183 year's El Niño. *Nature Geoscience*, 3(3), 168-172.
- 1184 Izumo, T., Lengaigne, M., Vialard, J., Luo, J. J., Yamagata, T., & Madec, G. (2014). Influence
1185 of Indian Ocean Dipole and Pacific recharge on following year's El Niño: interdecadal
1186 robustness. *Climate dynamics*, 42(1-2), 291-310.
- 1187 Izumo, T., Vialard, J., Dayan, H., Lengaigne, M., & Suresh, I. (2016). A simple estimation of
1188 equatorial Pacific response from windstress to untangle Indian Ocean Dipole and Basin
1189 influences on El Niño. *Climate dynamics*, 46(7-8), 2247-2268.
- 1190 Izumo, T., Khodri, M., Lengaigne, M., & Suresh, I. (2018). A subsurface Indian Ocean dipole
1191 response to tropical volcanic eruptions. *Geophysical Research Letters*, 45(17), 9150–
1192 9159. <https://doi.org/10.1029/2018GL078515>
- 1193 Izumo, T., Lengaigne, M., Vialard, J., Suresh, I., & Planton, Y. (2019). On the physical
1194 interpretation of the lead relation between Warm Water Volume and the El Niño
1195 Southern Oscillation. *Climate Dynamics*, 52(5), 2923-2942.
- 1196 Izumo, T., Vialard, J., Lengaigne, M., & Suresh, I. (2020). Relevance of relative sea surface
1197 temperature for tropical rainfall interannual variability. *Geophysical Research
1198 Letters*, 47(3), e2019GL086182.
- 1199 Izumo, T., & Colin, M. (2022). Improving and harmonizing El Niño recharge indices.
1200 *Geophysical Research Letters*, 49, e2022GL101003. [https://doi.
1201 org/10.1029/2022GL101003](https://doi.org/10.1029/2022GL101003)
- 1202 Jin, F. F. (1997a). An equatorial ocean recharge paradigm for ENSO. Part I: Conceptual
1203 model. *Journal of the atmospheric sciences*, 54(7), 811-829.
- 1204 Jin, F. F. (1997b). An equatorial ocean recharge paradigm for ENSO. Part II: A stripped-down
1205 coupled model. *Journal of Atmospheric Sciences*, 54(7), 830-847.
- 1206 Jin, F. F., & An, S. I. (1999). Thermocline and zonal advective feedbacks within the equatorial
1207 ocean recharge oscillator model for ENSO. *Geophysical research letters*, 26(19), 2989-
1208 2992.
- 1209 Jin, F.- F., Lin, L., Timmermann, A., & Zhao, J. (2007). Ensemble- mean dynamics of the
1210 ENSO recharge oscillator under state- dependent stochastic forcing. *Geophysical
1211 Research Letters*, 34(3), L03807. <https://doi.org/10.1029/2006GL027372>

- 1212 Jin, F. F., Chen, H. C., Zhao, S., Hayashi, M., Karamperidou, C., Stuecker, M. F., ... & Geng,
1213 L. (2020). Simple ENSO models. *El Niño Southern Oscillation in a Changing Climate*,
1214 119-151.
- 1215 Johnson, N. C., & Xie, S.- P. (2010). Changes in the sea surface temperature threshold for
1216 tropical convection. *Nature Geoscience*, 3(12), 842–845.
1217 <https://doi.org/10.1038/ngeo1008>
- 1218 Johnson, N. C., & Kosaka, Y. (2016). The impact of eastern equatorial Pacific convection on
1219 the diversity of boreal winter El Niño teleconnection patterns. *Climate Dynamics*, 47(12),
1220 3737-3765.
- 1221 Jourdain NC, Lengaigne M, Vialard J, Izumo T, Gupta AS (2016) Fur- ther insights on the
1222 influence of the Indian Ocean dipole on the following year’s ENSO from observations
1223 and CMIP5 models. *J Clim* 29(2):637–658. <https://doi.org/10.1175/JCLI-D-15-0481.1>
- 1224 Keane, A., Krauskopf, B., & Postlethwaite, C. (2016). Investigating irregular behavior in a
1225 model for the El Niño Southern Oscillation with positive and negative delayed
1226 feedback. *SIAM Journal on Applied Dynamical Systems*, 15(3), 1656-1689.
- 1227 Kosaka, Y., & Xie, S. P. (2013). Recent global-warming hiatus tied to equatorial Pacific
1228 surface cooling. *Nature*, 501(7467), 403-407.
- 1229 Kug, J. S., & Kang, I. S. (2006). Interactive feedback between ENSO and the Indian
1230 Ocean. *Journal of climate*, 19(9), 1784-1801.
- 1231 Kug, J. S., Jin, F. F., & An, S. I. (2009). Two types of El Niño events: cold tongue El Niño and
1232 warm pool El Niño. *Journal of climate*, 22(6), 1499-1515.
- 1233 Kug, J.-S., Choi, J., An, S.-I., Jin, F.-F., and Wittenberg, A. T. (2010). Warm Pool and Cold
1234 Tongue El Niño Events as Simulated by the GFDL 2.1 Coupled GCM. *J. Clim.* 23,
1235 1226–1239. doi:10.1175/2009JCLI3293.1
- 1236 Lai AWC, Herzog M, Graf HF (2015) Two key parameters for the El Niño continuum: zonal
1237 wind anomalies and Western Pacific sub- surface potential temperature. *Clim Dyn*
1238 45(11–12):3461–3480. <https://doi.org/10.1007/s00382-015-2550-0>
- 1239 Lengaigne M, Boulanger J-P, Menkes C, Spencer H (2006) Influence of the seasonal cycle on
1240 the termination of El Niño events in a coupled general circulation model. *J Clim*
1241 19(9):1850–1868. <https://doi.org/10.1175/JCLI3706.1>
- 1242 Linz M, Tziperman E, MacMartin DG (2014) Process-based analysis of climate model ENSO
1243 simulations: intermodel consistency and compensating errors. *J Geophys Res* 119:7396–
1244 7409. <https://doi.org/10.1002/2013JD021415>

- 1245 Lloyd J, Guilyardi E, Weller H (2010) The role of atmosphere feed- backs during ENSO in the
1246 CMIP3 models. Part II: using AMIP runs to understand the heat flux feedback
1247 mechanisms. *Clim Dyn* 37:1271–1292. <https://doi.org/10.1007/s00382-010-0895-y>
- 1248 Lu, B., Jin, F. F., & Ren, H. L. (2018). A coupled dynamic index for ENSO
1249 periodicity. *Journal of Climate*, 31(6), 2361-2376.
- 1250 Luo J-J, Zhang R, Behera S, Masumoto Y, Jin F-F, Lukas R, Yamagata T (2010) Interaction
1251 between El Niño and extreme Indian Ocean Dipole. *J Clim* 23:726–742
- 1252 McCreary, J. P., & Lu, P. (1994). Interaction between the subtropical and equatorial ocean
1253 circulations: The subtropical cell. *Journal of Physical Oceanography*, 24(2), 466-497.
- 1254 McPhaden, M. J. (2012). A 21st century Shift in the Relationship between ENSO SST and
1255 Warm Water Volume Anomalies. *Geophys. Res. Lett.* 39, a–n. doi:10.
1256 1029/2012GL051826
- 1257 Meinen CS, McPhaden MJ (2000) Observations of warm water volume changes in the
1258 equatorial Pacific and their relationship to El Niño and La Niña. *J Clim* 13:3551–3559
- 1259 Menkes CE, Lengaigne M, Marchesiello P, Jourdain NC, Vincent EM, Lefèvre J, Chauvin F,
1260 Royer JF. Comparison of tropical cyclogenesis indices on seasonal to interannual
1261 timescales. *Climate dynamics*. 2012 Jan;38:301-21.
- 1262 Neelin, J. D., Battisti, D. S., Hirst, A. C., Jin, F. F., Wakata, Y., Yamagata, T., & Zebiak, S. E.
1263 (1998). ENSO theory. *Journal of Geophysical Research: Oceans*, 103(C7), 14261-
1264 14290.
- 1265 Okumura, Y. M. (2019). ENSO diversity from an atmospheric perspective. *Current Climate
1266 Change Reports*, 5(3), 245-257.
- 1267 Picaut, J., Ioualalen, M., Menkès, C., Delcroix, T., & Mcphaden, M. J. (1996). Mechanism of
1268 the zonal displacements of the Pacific warm pool: Implications for
1269 ENSO. *Science*, 274(5292), 1486-1489.
- 1270 Picaut J, Masia F, du Penhoat Y (1997) An advective–reflective conceptual model for the
1271 oscillatory nature of the ENSO. *Science* 277:663–666.
1272 <https://doi.org/10.1126/science.277.5326.663>
- 1273 Planton, Y., Vialard, J., Guilyardi, E., Lengaigne, M., & Izumo, T. (2018). Western Pacific
1274 oceanic heat content: A better predictor of La Niña than of El Niño. *Geophysical
1275 Research Letters*, 45(18), 9824-9833.
- 1276 Power, S. B. (2011). Simple analytic solutions of the linear delayed-action oscillator equation
1277 relevant to ENSO theory. *Theoretical and applied climatology*, 104(1), 251-259.

- 1278 Praveen Kumar, B., Jérôme Vialard, Matthieu Lengaigne, V. S. N. Murty, Michael J.
1279 Mcphaden, M. F. Cronin, Françoise Pinsard, and K. Gopala Reddy. "TropFlux wind
1280 stresses over the tropical oceans: evaluation and comparison with other
1281 products." *Climate dynamics* 40 (2013): 2049-2071.
- 1282 Giese, B. S., & Ray, S. (2011). El Niño variability in simple ocean data assimilation (SODA),
1283 1871–2008. *Journal of Geophysical Research: Oceans*, 116(C2).
- 1284 Ren, H.-L., and Jin, F.-F. (2013). Recharge Oscillator Mechanisms in Two Types of ENSO. *J.*
1285 *Clim.* 26, 6506–6523. doi:10.1175/jcli-d-12-00601.1
- 1286 Ren, H., and R. Wang, 2020: Distinct Growth Rates of the Two ENSO Types. *Geophys. Res.*
1287 *Lett.*, 47, <https://doi.org/10.1029/2020GL088179>.
- 1288 Santoso, A., Mcphaden, M. J., & Cai, W. (2017). The defining characteristics of ENSO extremes
1289 and the strong 2015/2016 El Niño. *Reviews of Geophysics*, 55, 1079–1129. [https://doi.](https://doi.org/10.1002/2017RG000560)
1290 [org/10.1002/2017RG000560](https://doi.org/10.1002/2017RG000560)
- 1291 Shin, N. Y., Kug, J. S., Stuecker, M. F., Jin, F. F., Timmermann, A., & Kim, G. I. (2022). More
1292 frequent central Pacific El Niño and stronger eastern pacific El Niño in a warmer
1293 climate. *npj Climate and Atmospheric Science*, 5(1), 101.
- 1294 Srinivas G, Vialard J, Lengaigne M, Izumo T, Guilyardi E. Relative contributions of sea surface
1295 temperature and atmospheric nonlinearities to ENSO asymmetrical rainfall response.
1296 *Journal of Climate*. 2022 Jun 15;35(12):3725-45.
- 1297 Srinivas G, Vialard J, Liu F, Voltaire A, Izumo T, Guilyardi E, Lengaigne M,. Dominant
1298 contribution of atmospheric non-linearities to ENSO asymmetry and extreme El Niño
1299 events. *Sci. Reports*, under revision, 2024.
- 1300 Stellema A, Sen Gupta A, Taschetto AS and Feng M (2022) Pacific Equatorial Undercurrent:
1301 Mean state, sources, and future changes across models. *Front. Clim.* 4:933091. doi:
1302 10.3389/fclim.2022.933091
- 1303 Stuecker, M. F., Timmermann, A., Jin, F. F., McGregor, S., & Ren, H. L. (2013). A
1304 combination mode of the annual cycle and the El Niño/Southern Oscillation. *Nature*
1305 *Geoscience*, 6(7), 540-544.
- 1306 Suarez MJ, Schopf PS (1988) A delayed action oscillator for ENSO. *J Atmos Sci* 45:3283–
1307 3287
- 1308 Sullivan A, Luo JJ, Hirst AC, Bi D, Cai W, He J. Robust contribution of decadal anomalies to
1309 the frequency of central-Pacific El Niño. *Scientific reports*. 2016 Dec 5;6(1):38540.

- 1310 Takahashi K, Montecinos A, Goubanova K, Dewitte B (2011) ENSO regimes: reinterpreting
1311 the canonical and Modoki El Niño. *Geo-phys Res Lett* 38:1–5.
1312 <https://doi.org/10.1029/2011GL047364>
- 1313 Takahashi, K., Karamperidou, C., & Dewitte, B. (2019). A theoretical model of strong and
1314 moderate El Niño regimes. *Climate Dynamics*, 52(12), 7477–7493.
1315 <https://doi.org/10.1007/s00382-018-4100-z>
- 1316 Timmermann, Axel, et al. "El Niño–southern oscillation complexity." *Nature* 559.7715 (2018):
1317 535-545.
- 1318 Tippett MK, Camargo SJ, Sobel A (2011) A poisson regression index for tropical cyclone
1319 genesis and the role of large-scale vorticity in genesis. *J clim.*
1320 [doi:10.1175/2010JCLI3811.1](https://doi.org/10.1175/2010JCLI3811.1), accepted
- 1321 Thual, S. and Dewitte, B., 2023: The role of spatial shifting in El Niño/Southern Oscillation
1322 complexity, accepted.
- 1323 Thual, S., B. Dewitte, N. Ayoub, and O. Thual, 2013: An Asymptotic Expansion for the
1324 Recharge-Discharge Model of ENSO, *J. Phys. Oceanogr.*, 43, 1407–1416.
- 1325 Trenberth KE, Stepaniak DP. Indices of el Niño evolution. *Journal of climate*. 2001 Apr
1326 15;14(8):1697-701.
- 1327 Tziperman, E., Cane, M. A., & Zebiak, S. E. (1995). Irregularity and locking to the seasonal
1328 cycle in an ENSO prediction model as explained by the quasi-periodicity route to
1329 chaos. *Journal of Atmospheric Sciences*, 52(3), 293-306.
- 1330 Van Oldenborgh, G. J., Hendon, H., Stockdale, T., L’Heureux, M., De Perez, E. C., Singh, R.,
1331 & Van Aalst, M. (2021). Defining El Niño indices in a warming climate. *Environmental*
1332 *Research Letters*, 16(4), 044003.
- 1333 Vecchi, G. A., & Soden, B. J. (2007). Effect of remote sea surface temperature change on
1334 tropical cyclone potential intensity. *Nature*, 450(7172), 1066–1070.
1335 <https://doi.org/10.1038/nature06423>
- 1336 Vialard, J., Menkes, C., Boulanger, J. P., Delecluse, P., Guilyardi, E., McPhaden, M. J., &
1337 Madec, G. (2001). A model study of oceanic mechanisms affecting equatorial Pacific sea
1338 surface temperature during the 1997–98 El Niño. *Journal of Physical*
1339 *Oceanography*, 31(7), 1649-1675.
- 1340 Vijayeta, A., and D. Dommenges, 2018: An evaluation of ENSO dynamics in CMIP
1341 simulations in the framework of the recharge oscillator model. *Clim Dyn*, 51, 1753-
1342 1771, <https://doi.org/10.1007/s00382-017-3981-6>.

- 1343 Vincent, E. M., Lengaigne, M., Menkes, C. E., Jourdain, N. C., Marchesiello, P., & Madec, G.
1344 (2011). Interannual variability of the South Pacific Convergence Zone and implications
1345 for tropical cyclone genesis. *Climate dynamics*, 36, 1881-1896.
- 1346 Wang, C., & Picaut, J. (2004). Understanding ENSO physics—A review. *Earth's Climate: The*
1347 *Ocean–Atmosphere Interaction, Geophys. Monogr*, 147, 21-48.
- 1348 Wang, C. (2001). A unified oscillator model for the El Niño–Southern Oscillation. *Journal of*
1349 *Climate*, 14(1), 98-115.
- 1350 Wang, R., and H.-L. Ren, 2017: The linkage between two ENSO types/modes and the
1351 interdecadal changes of ENSO around the year 2000. *Atmospheric and Oceanic Science*
1352 *Letters*, 10, 168-174, <https://doi.org/10.1080/16742834.2016.1258952>.
- 1353 Williams, I. N., & Patricola, C. M. (2018). Diversity of ENSO events unified by convective
1354 threshold sea surface temperature: A nonlinear ENSO index. *Geophysical Research*
1355 *Letters*, 45(17), 9236-9244.
- 1356 Wyrski K (1985) Water displacements in the Pacific and the genesis of El Niño cycles. *J*
1357 *Geophys Res* 90:7129–7132
- 1358 Xie R, Huang F, Jin FF, Huang J. The impact of basic state on quasi-biennial periodicity of
1359 central Pacific ENSO over the past decade. *Theoretical and Applied Climatology*. 2015
1360 Apr;120:55-67.
- 1361 Yu et al. (2015; they combine a recharge oscillator to a slab ocean coupled to an AGCM to
1362 capture ENSO dynamics and diversity
- 1363 Zeller M, McGregor S, Spence P. Hemispheric asymmetry of the Pacific shallow meridional
1364 overturning circulation. *Journal of Geophysical Research: Oceans*. 2019
1365 Aug;124(8):5765-86
- 1366 Zeller M, McGregor S, van Sebille E, Capotondi A, Spence P. Subtropical-tropical pathways
1367 of spiciness anomalies and their impact on equatorial Pacific temperature. *Climate*
1368 *Dynamics*. 2021 Feb;56(3-4):1131-44.
- 1369 Zhang, W., Li, S., Jin, F.- F., Xie, R., Liu, C., Stuecker, M. F., & Xue, A. (2019). ENSO regime
1370 changes responsible for decadal phase relationship variations between ENSO sea surface
1371 temperature and warm water volume. *Geophysical Research Letters*, 46, 7546–7553.
1372 <https://doi.org/10.1029/2019GL082943>
- 1373 Zhu, X., Greatbatch, R. J., & Claus, M. (2017). Interannual variability of tropical Pacific sea
1374 level from 1993 to 2014. *Journal of Geophysical Research: Oceans*, 122(1), 602-616.

Supplementary Information for
The hybrid Recharge Delayed Oscillator: a more realistic El Niño
conceptual model

Takeshi Izumo,^{a,b} Maxime Colin,^{c,d,e} Fei-Fei Jin,^f and Bastien Pagli,^a

^a *Institut de Recherche pour le Développement (IRD), UMR241 SECOPOL (ex-EIO) laboratory, Université de la Polynésie Française (UPF), Tahiti, French Polynesia*

^b *formerly at IRD, Sorbonne Université - CNRS-IRD-MNHN, LOCEAN Laboratory, IPSL, Paris, France*

^c *Climate Change Research Centre, University of New South Wales, Sydney, New South Wales, Australia*

^d *Laboratoire GEPASUD, University of French Polynesia, Tahiti, French Polynesia*

^e *now at Leibniz Centre for Tropical Marine Research (ZMT), Bremen, Germany*

^f *Department of Atmospheric Sciences, University of Hawai'i at Mānoa, Honolulu, HI, United States*

Submitted to Journal of Climate on March 3rd, 2023

Revised, September 6th, 2023

Revised, January 18th, 2024

Corresponding author: Takeshi Izumo, takeshi.izumo@ird.fr

Supplementary text S1: Additional data and methods information

a. Validation of indices (sensitivity tests)

Using Niño3 region instead of Niño3.4 region leads to qualitatively-similar results. It leads to only a weak decrease for RDO temperature equation skill (see RDO equations in section 3), and a slight F_1 increase, for instance in obs2 from 0.11 to 0.12 ± 0.02 month⁻¹ and a β decrease from 0.09 to 0.07 ± 0.02 month⁻¹ as expected physically because Niño3 is slightly further east (cf. Fig. 2a and Suppl. Fig. S1).

For the recharge index, the debate is partly caused by the potentially-misleading character of the most commonly used recharge index, the equatorial Pacific OHC index, h_{eq} , which mixes the slow recharge mode with the temporary rapid variations related to equatorial Kelvin waves that project strongly onto h_{eq} , as shown by Izumo et al. (2018a). Through an objective approach to optimize the fit of the RO differential equations with observations, Izumo and Colin (2022) have thus developed a more physical and optimized index of the ENSO slow recharge mode, uncorrelated to T_E (i.e. independent of the fast adjustment mode). They separate the fast oceanic adjustment in phase with T_E (tilt mode) and the slow oceanic component out of phase with T_E , i.e. the pure recharge/discharge process (whose tendency is forced by in-phase currents associated to T_E). Doing this separation allows them to reconcile western and equatorial recharge indices, as well as SLA, Z20 or OHC-based indices, as all the recharge indices become much more similar when only looking at their pure recharge components independent of T_E . The study also investigates what is the best averaging region optimizing the RO set of equations. The optimal two-rectangle region to capture the slow recharge process is obtained by adding a h_{sw} southwest Pacific box to h_{eq} box (see Fig. 1b that physically represents the recharge after long-lasting easterly anomalies in observations; see Iwakiri and Watanabe 2022 for processes driving long-lasting La Niña). This recharge (in the case of easterly anomalies, discharge in the westerly case) is through (1) an equatorial basin adjustment with the leakage of OHC negative anomalies towards the poles along the eastern boundary recharging equatorial OHC (e.g. Izumo et al. 2018, their Supplementary Figure S4) and (2) meridionally-asymmetric Ekman pumping recharging more the southwest Pacific, that will progressively favor positive T_E and thus El Niño onset (this asymmetry explains why the optimal region, “eq+sw”, is asymmetric). Adding “sw” to “eq” further improves the fit of RO equations and ENSO hindcasts, with the added advantage of reducing the correlation to T_E . Note that this eq+sw region is actually also optimal for the RDO set of equations developed in the present study (not shown). Note also that one

could also use the 20°C isotherm depth, the Z20, instead of SLA, but the SLA is directly observed by satellite and more easily available in CMIP database and gives equivalent results. In fine, the optimized recharge index statistically-independent from T_E is $h_{ind_eq+sw} = h_{eq+sw} - K T_E \approx h_{eq+sw}$ (Izumo and Colin 2022).

Indeed, the RDO equations 6 and 7 remain in practice quasi-similar if we use h_{eq+sw} instead of h_{ind_eq+sw} for the recharge index. As K is so small, the shared variance between h_{eq+sw} and T_E is for instance only 9% for obs2. Hence i.e. β (e.g. $0.09 \pm 0.02 \text{ month}^{-1}$ in obs2) remains mathematically identical. F_1 does not change at the second digit, remaining for instance in obs2 at $0.11 \pm 0.02 \text{ month}^{-1}$. F_2 changes weakly, from 0.16 ± 0.02 to $0.17 \pm 0.02 \text{ month}^{-1}$. Only R and ε decrease, R from 0.03 ± 0.02 to $0.00 \pm 0.02 \text{ month}^{-1}$ and ε from 0.00 ± 0.02 to $-0.05 \pm 0.02 \text{ month}^{-1}$ in obs2 for instance. The correlation skill for the temperature equation remains mathematically identical, and the skill for recharge equation decreases slightly, e.g. from 0.68 to 0.66 in obs2.

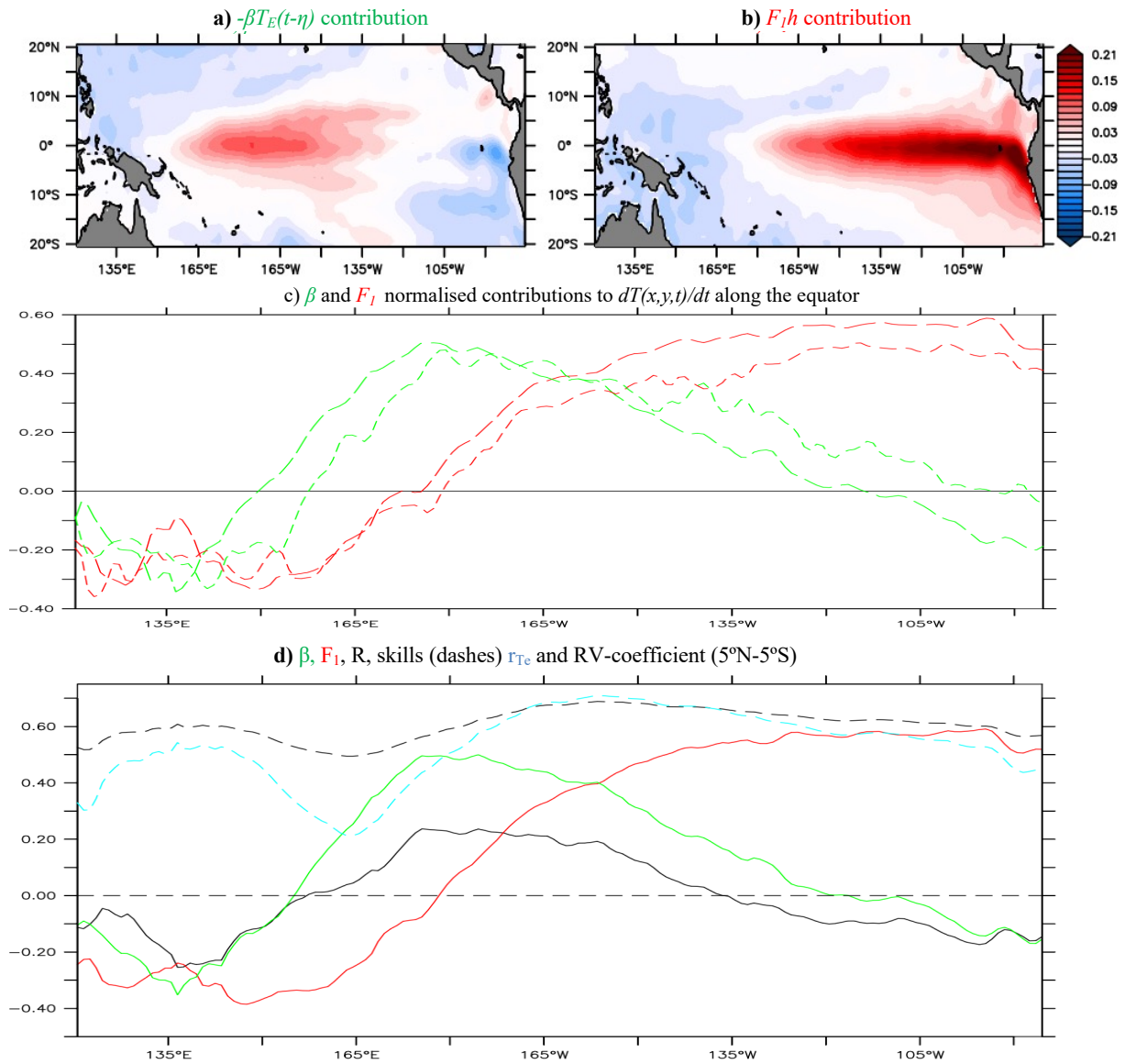
b. Methods

Here we use typical statistical methods. The monthly seasonal cycle (computed by averaging each month of the year over the whole period) is removed and intraseasonal variations are filtered out by a 4-month window Hanning filter, so that periods lower than ~ 2 -month are removed (considering as fast adjustments anything with timescales less than ~ 2 -month). For the long historical datasets ‘obs2’ (ORAS5 analysis over its whole extended period 1959-2018 + HadiSST), hist (SODA2.2.6 + HadiSST, note: at the time of doing the present research work, CERA20C ECMWF server was unfortunately down, and could not be used) and CMIP, we use in addition to the intraseasonal low-pass filter a high-pass Hanning filter (14-year window) to remove interdecadal variability with cutting frequency at periods of ~ 10 -year, as our focus is here on ENSO main interannual timescale (note that interdecadal variability and possible trends are removed to be safe, but the results are quasi-similar without this high-pass filter).

Multivariate linear regressions are used to obtain the best-fit coefficients for the RDO differential equations. Note that for monthly values, it is important to use centered derivatives to estimate h and T_E tendencies accurately (using forward derivatives leads to biased parameters and thus biases of the RDO response to stochastic forcing). All along the paper, numbers are given for normalized h and T_E , i.e. divided by their standard deviation (STD), so as to represent typical amplitudes. Using formulae (30) of Bretherton et al. (1999), we have about one effective degree of freedom every 4 months, i.e. ~ 85 for obs1, ~ 140 for obs2, ~ 370 for hist, ~ 360 for

each CMIP5 model historical run, ~ 450 for each CMIP6 historical run. These effective numbers of degrees of freedom are sufficiently large, so that statistics are robust. For statistical significance, we can thus use typical two-tailed Student t-tests for e.g. 90% or 95% confidence intervals. As here, our effective number of degrees of freedom dof is sufficiently large, being always larger than ~ 80 , the reader can easily switch from 90 to 95% [or even 99%] intervals by multiplying by the ratio of the corresponding t-values, quite stable for dof above 30 (e.g. for $dof=80$, ratio of 1.2 to switch from 90 to 95% [ratio of 1.6 to switch from 90 to 99%]).

To obtain the RDO response to stochastic forcing, we use the Euler-Heun numerical scheme, necessary when a multiplicative noise is included. The stochastic forcings of the RDO are red noises (except for computing the spectral response, for which we force by white noise). Their amplitudes are based on the STD of the residuals of the tendency equations regression fits. We take a realistic decorrelation time of 5-day for these red noises, which we have estimated from the observed decorrelation time of the main stochastic forcing of ENSO, namely intraseasonal zonal windstress over the oceanic equatorial wave guide (2°N - 2°S) in the western Pacific (e.g. 120°E - 155°W) due to westerly wind bursts and to Madden-Julian Oscillation (MJO; using daily Tropflux product; Praveen Kumar et al. 2013; interannual anomalies removed with a 6-month window Hanning filter). This decorrelation time is qualitatively consistent with the one of Vijayeta and Dommenges (2018). Note that, at the end, the amplitude of the synthetic time series directly depends on this time. The larger the decorrelation time is, the more energy there will be at interannual timescales, the larger the RDO response will be. E.g. STD increases by about 0.3 from 5-day to 15-day, for non-linear seasonal RDO based on hist shown in Section 5 (even if the STD of the red noises are unchanged).



Suppl. Fig. S1. Contributions of $-\beta(x,y)T_E(t-\eta)$ and $F_1(x,y)h$ to $dT(x,y,t)/dt$. a and b) as Fig. 2ab but for obs1 (OISST+CMEMS satellite SLA, 1993-2021) instead of obs2 (HadiSST+ORAS5 SLA, 1959-2018), for verification. c) $\beta(x,y)$ (green) and $F_1(x,y)$ (red) along the equator (here normalized by dividing by $STD(dT(x,y,t)/dt)$ for a normalized comparison), for obs1 (long dashes) and obs2 (short dashes). d) as c, but for 5°N-5°S average for obs1 only, and by also showing R (black line) and the correlation skill for temperature tendency equation 8 in blue dashes, and the RV-coefficient skill for the system of the two tendency equations together (see definition in Izumo and Colin 2022). The β contribution is qualitatively close to central Pacific (CP) El Niño spatial pattern, and the F_1 contribution is closer to the eastern Pacific (EP) El Niño pattern, both in obs1 and obs2.

$-\beta T_E(t-\eta)$ contribution

$F_j h$ contribution

Temperature advection

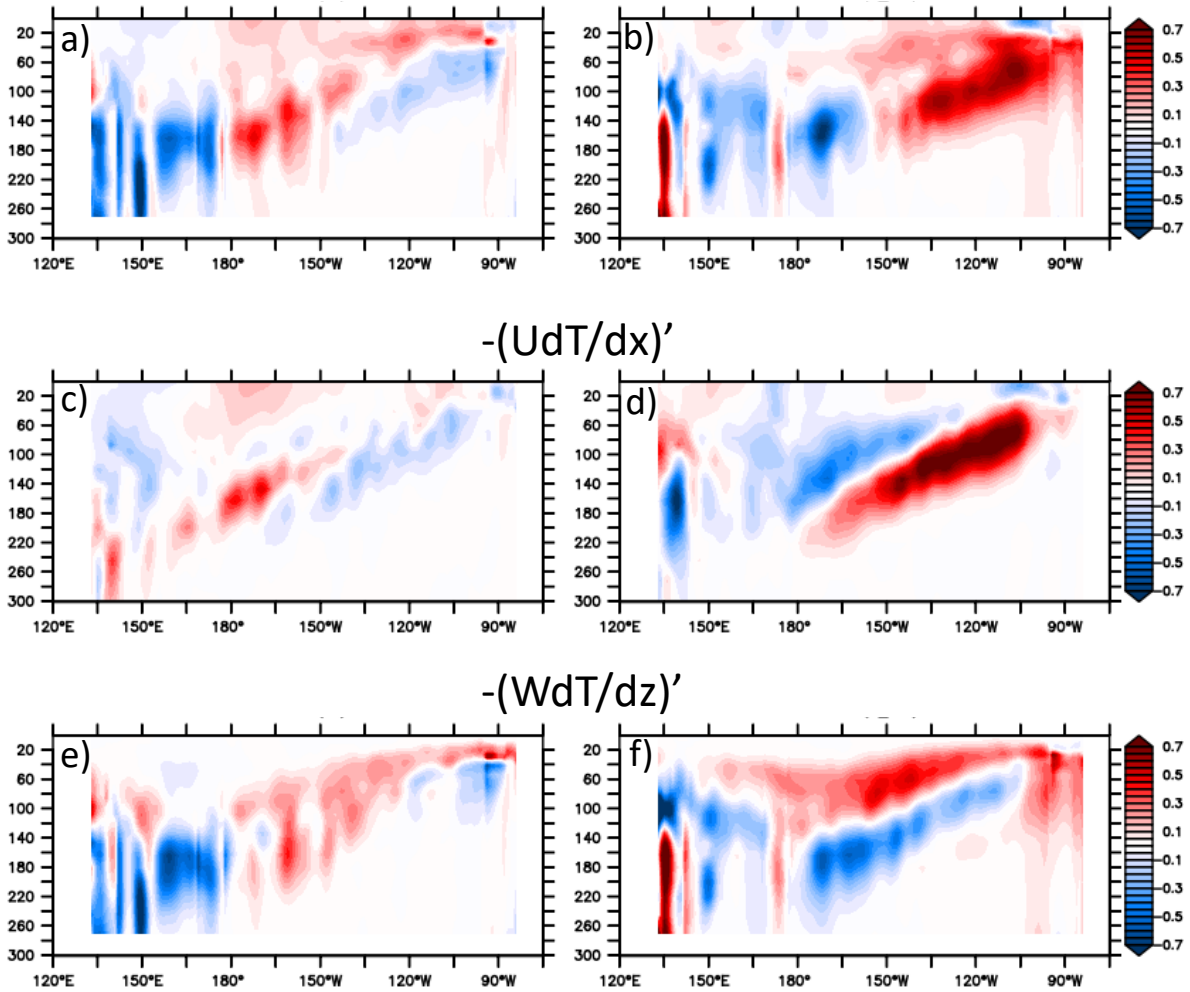
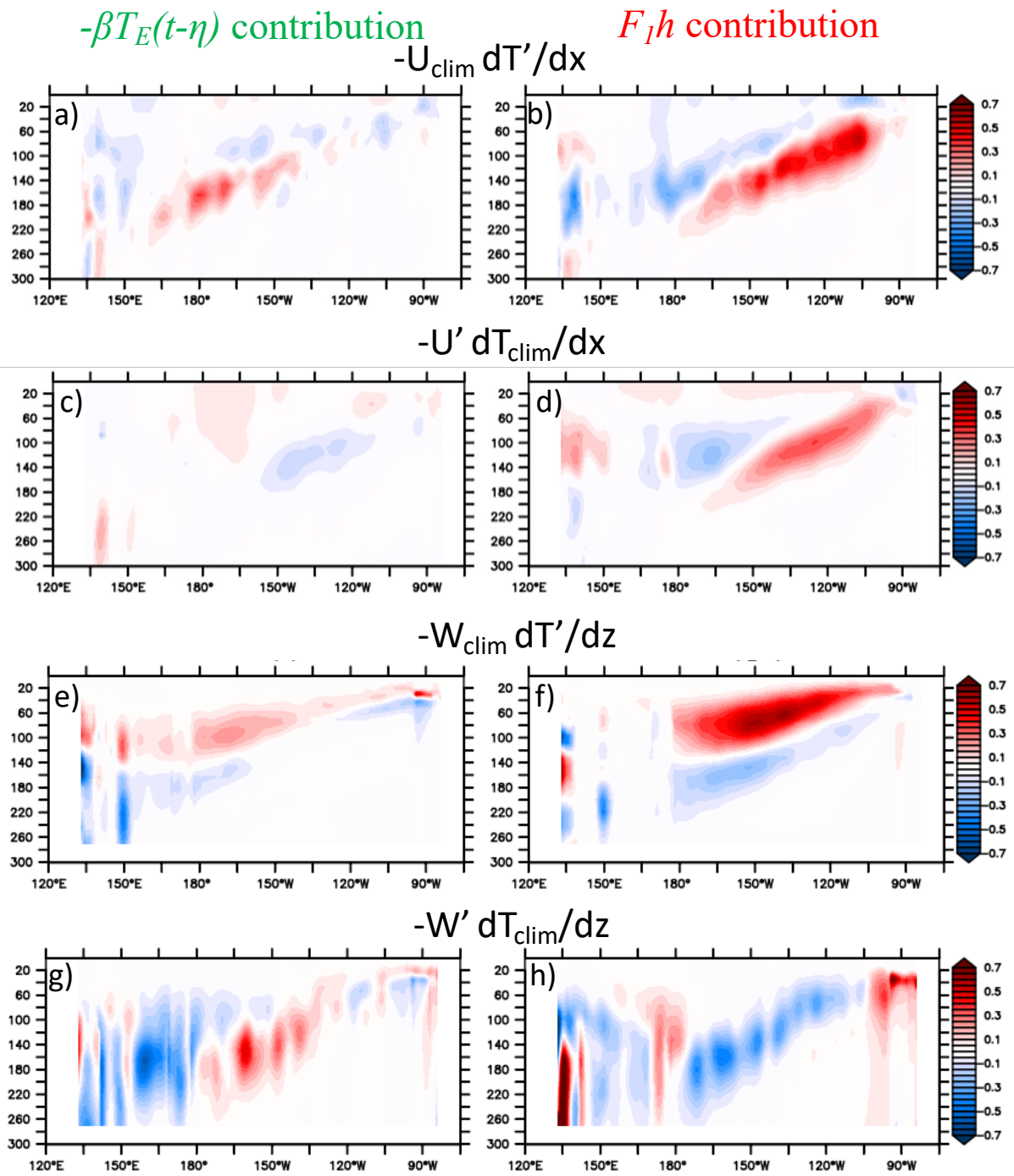
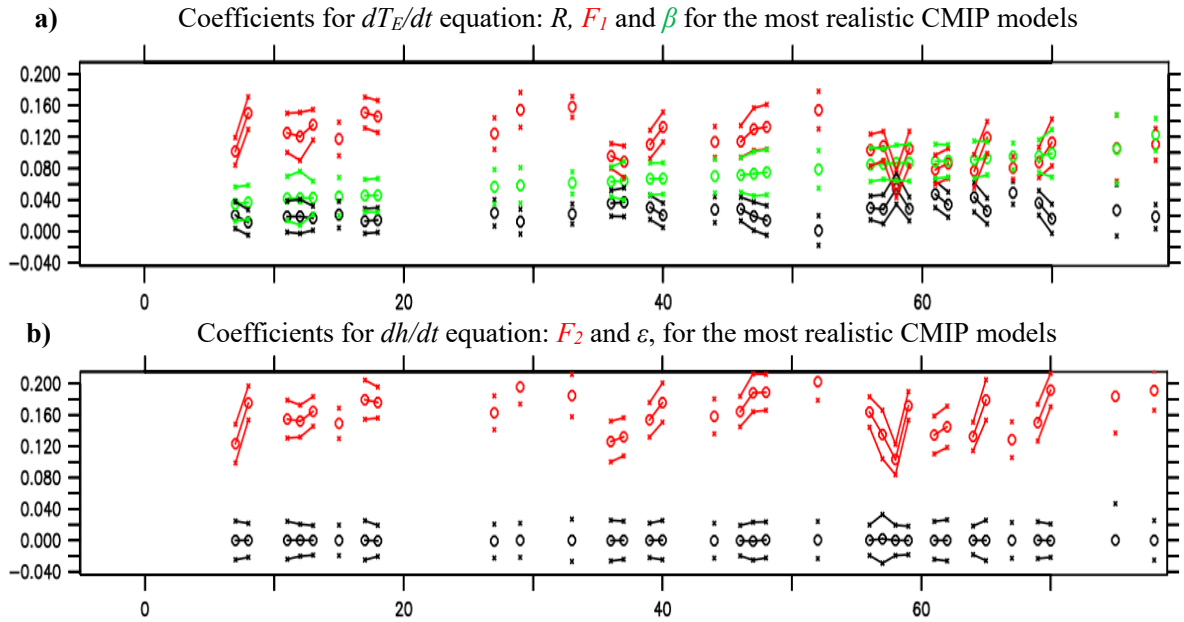


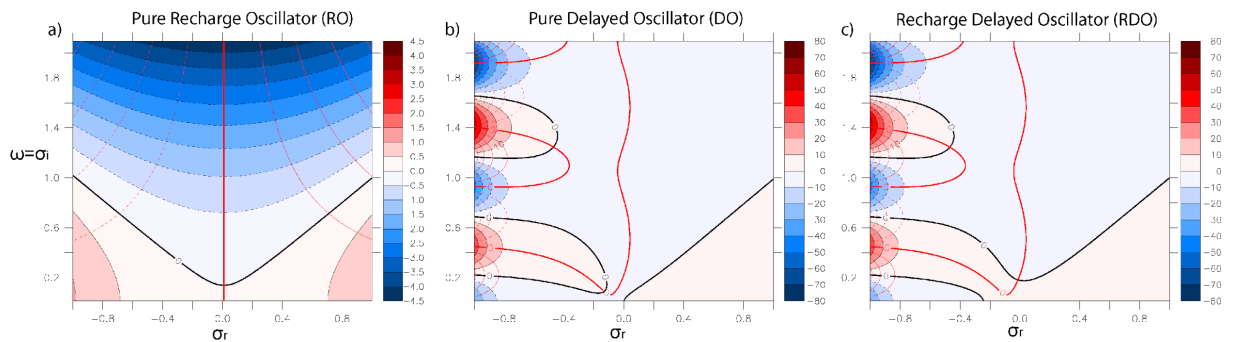
Fig. S2. Equatorial-depth section as Fig. 2efgh, but for a,b: the total heat advection $-U\partial T/\partial x - V\partial T/\partial y - W\partial T/\partial z$ (for monthly ORAS5, K month^{-1}); **c,d:** zonal advection $-U\partial T/\partial x$; **e,f:** vertical advection $-W\partial T/\partial z$ (sign convention: z increases upward (i.e. if e.g. $z=-50$ at depth 50m) and W is positive upward). The role of $-V\partial T/\partial y$ is not represented here, as it is negligible (Tropical Instability Waves (TIWs) damping effect is here not taken into account, as we only consider here monthly fields, in which TIWs are strongly filtered out). A 8° -window Hanning filter has been applied along longitude to reduce spatial noise before plotting. To interpret Fig. S2 correctly, remember that T' has much more variance around the thermocline, as even relatively-weak anomalies of the thermocline depth lead to large T' anomalies. Thus, the largest signals here are around the thermocline. Yet the weaker signals near the surface are actually essential to understand SST (and RSST) tendency (e.g. in the central Pacific on panels a and c) and should not be underestimated: they are weak in absolute amplitude but can be large in % of explained variance.



Suppl. Fig. S3. As Fig. S2, but here decomposing the zonal and vertical advections $-U\partial T/\partial x$ and $-W\partial T/\partial z$ into the contributions of anomalous current advecting climatological temperature gradient, and of anomalous temperature gradient advected by climatological current.

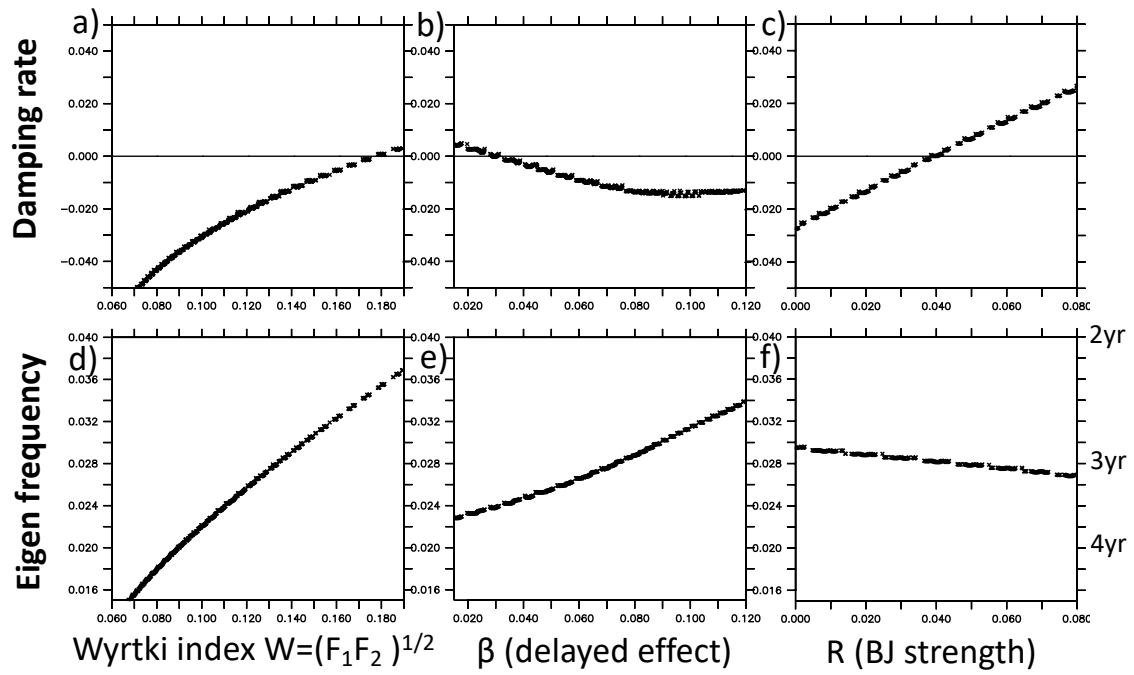


Suppl. Fig. S4. As Fig. 4 (keeping the same sorting by β), but only for the selected most realistic CMIP models in terms of ENSO STD and ENSO spectral shape. Both criteria have to be between 3/4 and 4/3 of their observed values (that are $\text{STD} \approx 0.8\text{K}$ and spectral shape ≈ 1.2). Interestingly, the models with the lowest β values (on the left) are rejected, as well as most models with the largest β values (on the right). The MMM of the RDO parameters for selected models ($R=0.0259$, $F_1=0.107$, $\beta=0.072$, $F_2=0.161$, $\varepsilon=0.00 \text{ month}^{-1}$) are almost similar to the MMM for all models ($R=0.0249$, $F_1=0.110$, $\beta=0.068$, $F_2=0.152$, $\varepsilon=0.00 \text{ month}^{-1}$). Note the slight increases in the ratio β/F_1 and in F_2 for the MMM of the selected models.



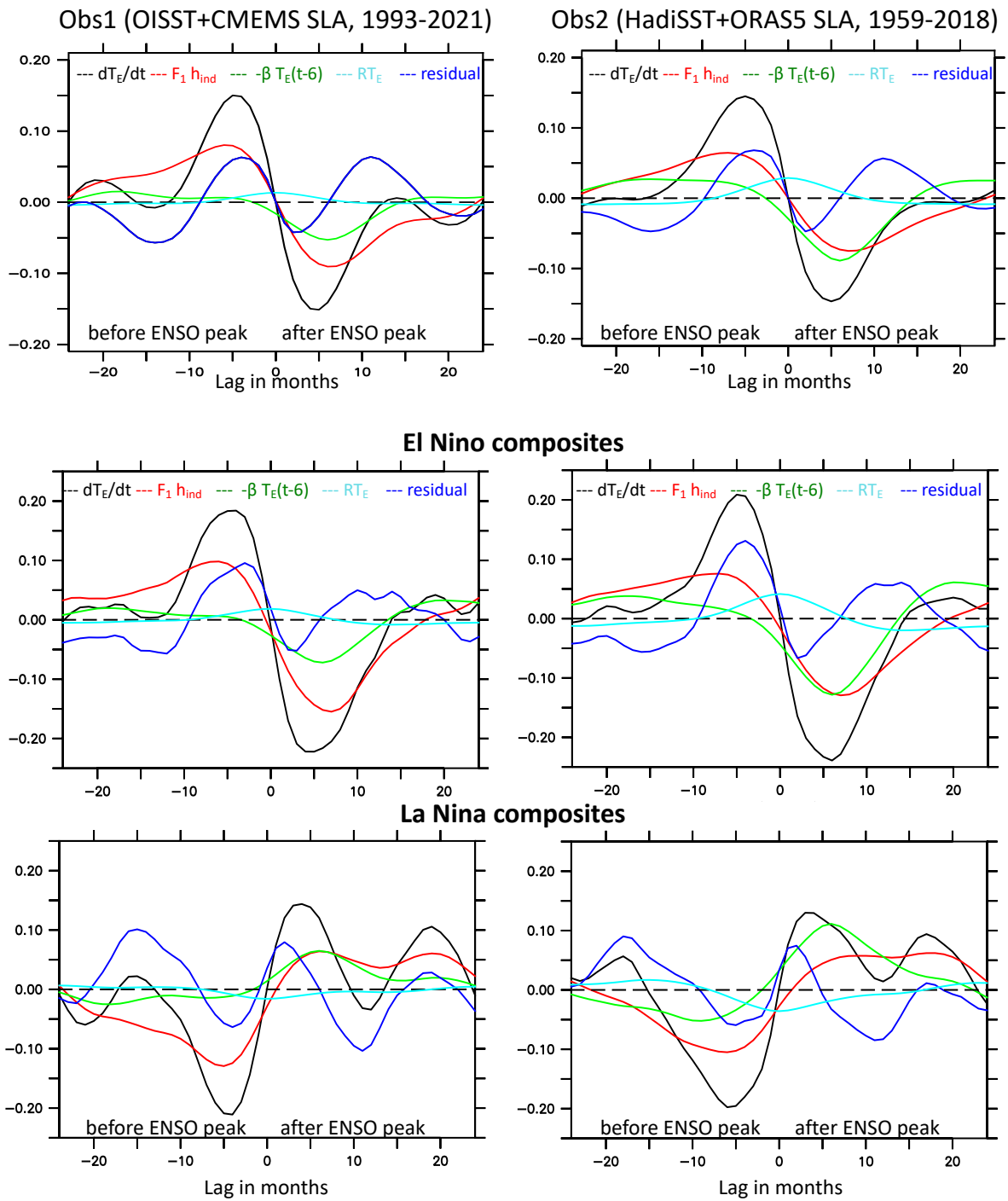
Suppl. Fig. S5. Resolution of the RDO differential equations system for solutions in $e^{\sigma t}$, by finding its eigenvalues σ which obey the characteristic equation (10): $\sigma^2 - R\sigma + \beta\sigma e^{-\sigma\tau} + F_1F_2 = 0$. Real part of the left-hand side term of equation (10) in color shading and black contours, imaginary part in red contours, as a function of σ_r along the x-axis and of σ_i along the y-axis (with $\sigma = \sigma_r + i\sigma_i$, its real part σ_r being the exponential growth - or damping rate if negative - and its imaginary part σ_i being the angular frequency ω , here in rad month^{-1} ; $0.2 \text{ rad month}^{-1}$ corresponds to a period of 31.4 month). The solutions of the equation, i.e. the eigenvalues, are where both real and imaginary parts are zero, i.e. where the black and red zero-line contours intersect. Panel a is for $\beta = 0$ case (pure RO), panel b is for $F_1 = 0$ case (pure DO) and panel c is for the RDO. The RDO has an infinity of eigenmodes due to the delayed term, like the DO, while the RO only has one. RDO 1st mode is of low-frequency (period ≈ 3 years) and weakly-damped (damping rate $\approx 0.02 \text{ month}^{-1} \approx 1/(4\text{year})$). RDO 2nd mode is of higher frequency (period ≈ 0.5 year) and is highly-damped (damping rate $\approx 0.45 \text{ month}^{-1} \approx 1/(2\text{months})$). But this 2nd mode still adds some high-frequency response to the RDO, absent in the pure RO (cf. spectra in Fig. 7b).

Dependency of eigenvalues to a single parameter

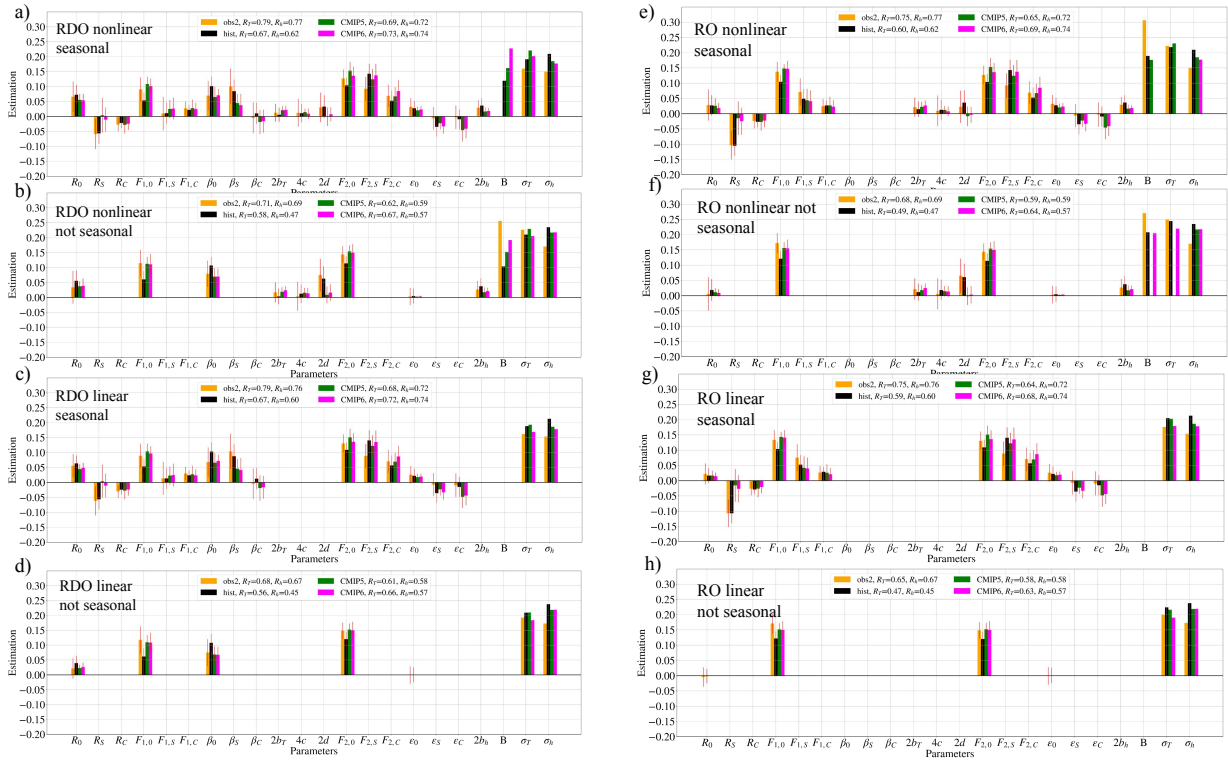


Suppl. Fig. S6. Dependency of eigenvalues to a single parameter. Dependency of RDO growth rate σ_r (upper panels) and eigenfrequency $f=\sigma_r/(2\pi)$ (lower panels) to Wyrтки angular frequency $W=(F_1F_2)^{1/2}$ (1st column), β (2nd column) and R (3rd column) separately (unit: month⁻¹).

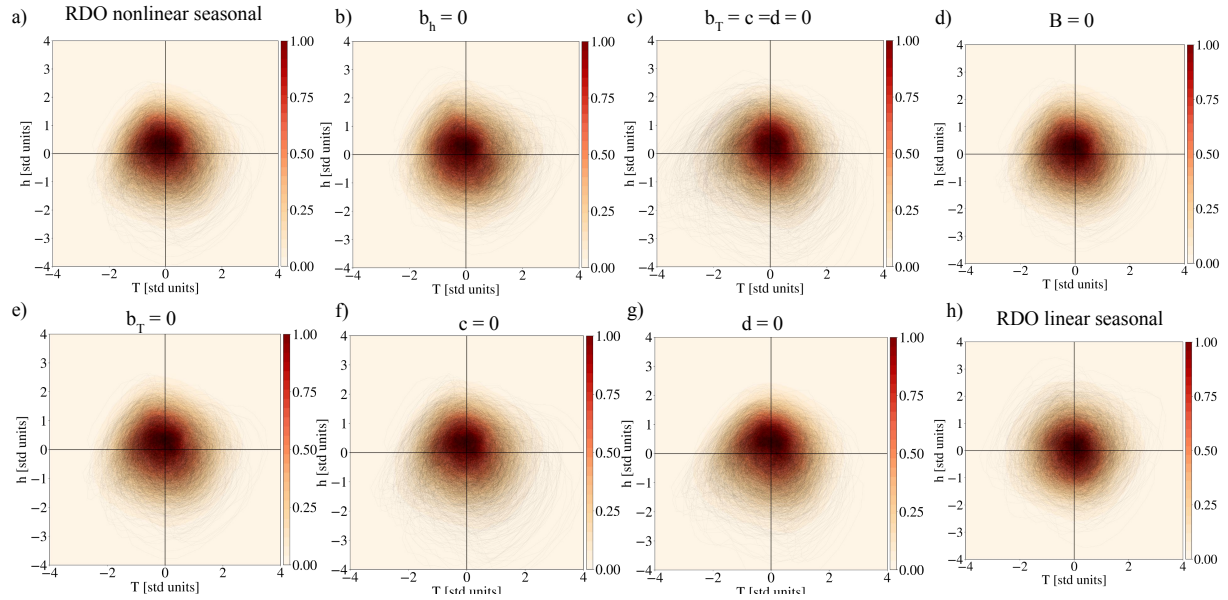
Observed lag-regression of the different terms of the dT_E/dt equation



Suppl. Fig. S7. Observed lag-regression (upper row) and lag-composites (middle and lower rows) of the different contributions to the T_E tendency equation. Upper row as Fig. 7b but for obs1 and obs2. In these observations, F_1 term dominates. The residual (i.e. noise plus non-linearities notably; cf. discussion) plays also a large role. The β term plays a large role after ENSO peak, but plays a weaker role compared to RDO synthetical timeseries before ENSO peak. Middle and lower rows show El Niño and La Niña separately (composites criteria: Niño3.4 RSST anomaly larger than two thirds of its standard deviation).



Suppl. Fig. S8. Comparison of the non-linear seasonal RDO to RO and their several versions. Bar plots as Fig. 8a, but here for both obs2 and hist, and for CMIP5 and CMIP6 separately.



Suppl. Fig. S9. Role of each non-linear term on RDO phase diagram. Comparison of the phase diagrams of the full non-linear seasonal RDO (based on hist; shown in panel a, which is the same as Fig. 9b) with specific RDO configurations in which some non-linear terms have been artificially set to 0, to illustrate their influences.

Table S1a. CMIP5 models historical runs used in the present study (as in Izumo et al. 2019). The 32 models, for which we have SST, SLA, precipitation and windstress available in their historical runs (at least over the 1861-2005 historical period, except CanCM4 starting in 1961).

Institute name	Model name
BCC	bcc-csm1-1
	bcc-csm1-1-m
CCCma	CanCM4
	CanESM2
CMCC	CMCC-CESM
	CMCC-CM
	CMCC-CMS
CNRM-CERFACS	CNRM-CM5
	CNRM-CM5-2
CSIRO-BOM	ACCESS1-0
	ACCESS1-3
CSIRO-QCCCE	CSIRO-Mk3-6-0
FIO	FIO-ESM
INM	inmcm4
IPSL	IPSL-CM5A-LR
	IPSL-CM5A-MR
	IPSL-CM5B-LR
LASG-CESS	FGOALS-g2
LASG-IAP	FGOALS-s2
MIROC	MIROC5
	MIROC-ESM
	MIROC-ESM-CHEM
MOHC	HadGEM2-CC
MRI	MRI-CGCM3
	MRI-ESM1
NASA-GISS	GISS-E2-R
	GISS-E2-R-CC
NCC	NorESM1-M
	NorESM1-ME
NOAA-GFDL	GFDL-CM2p1
	GFDL-CM3
	GFDL-ESM2G
	GFDL-ESM2M
NSF-DOE-NCAR	CESM1-CAM5

Table S1b. The 47 CMIP6 models historical runs used in the present study (with the reference number of the historical run used).

NCC_NorESM2-MM_gn_rlilp1fl_20191108
CCCma_CanESM5_gn_rlilp1fl_20190429
NCC_NorCPM1_gn_rlilp1fl_20200724
HAMMOZ-Consortium_MPI-ESM-1-2-HAM_gn_rlilp1fl_20190627
NOAA-GFDL_GFDL-CM4_gr_rlilp1fl_20180701
BCC_BCC-CSM2-MR_gn_rlilp1fl_20181126
MRI_MRI-ESM2-0_gr_rlilp1fl_20190904
INM_INM-CM5-0_gr1_rlilp1fl_20190610
EC-Earth-Consortium_EC-Earth3-Veg-LR_gn_rlilp1fl_20200919
NCAR_CESM2-WACCM_gr_rlilp1fl_20190808
NASA-GISS_GISS-E2-1-G_gn_rlilp1fl_20180827
AWI_AWI-ESM-1-1-LR_gn_rlilp1fl_20200212
E3SM-Project_E3SM-1-1_gr_rlilp1fl_20191204
THU_CIESM_gn_rlilp1fl_20200220
NASA-GISS_GISS-E2-1-H_gn_rlilp1fl_20190403
EC-Earth-Consortium_EC-Earth3-CC_gn_rlilp1fl_20210113
NASA-GISS_GISS-E2-1-G-CC_gn_rlilp1fl_20190815
NUIST_NESM3_gn_rlilp1fl_20190703
CMCC_CMCC-CM2-SR5_gn_rlilp1fl_20200616
EC-Earth-Consortium_EC-Earth3_gn_rlilp1fl_20200918
NCC_NorESM2-LM_gn_rlilp1fl_20190815
CSIRO-ARCCSS_ACCESS-CM2_gn_rlilp1fl_20191108
CAS_CAS-ESM2-0_gn_rlilp1fl_20200306
CAS_FGOALS-g3_gn_rlilp1fl_20191107
EC-Earth-Consortium_EC-Earth3-AerChem_gn_rlilp1fl_20200624
NCAR_CESM2_gr_rlilp1fl_20190308
MIROC_MIROC6_gn_rlilp1fl_20181212
NCAR_CESM2-FV2_gr_rlilp1fl_20191120
EC-Earth-Consortium_EC-Earth3-Veg_gn_rlilp1fl_20200919
CMCC_CMCC-ESM2_gn_rlilp1fl_20210114
CSIRO_ACCESS-ESM1-5_gn_rlilp1fl_20191115
E3SM-Project_E3SM-1-1-ECA_gr_rlilp1fl_20200127
BCC_BCC-ESM1_gn_rlilp1fl_20181129
NOAA-GFDL_GFDL-ESM4_gr_rlilp1fl_20190726
INM_INM-CM4-8_gr1_rlilp1fl_20190530
MPI-M_MPI-ESM1-2-HR_gn_rlilp1fl_20190710
E3SM-Project_E3SM-1-0_gr_rlilp1fl_20190826
MPI-M_MPI-ESM1-2-LR_gn_rlilp1fl_20190710
AWI_AWI-CM-1-1-MR_gn_rlilp1fl_20181218
NCAR_CESM2-WACCM-FV2_gr_rlilp1fl_20191120
CAM5_CAMS-CSM1-0_gn_rlilp1fl_20190708
FIO-QLNM_FIO-ESM-2-0_gn_rlilp1fl_20191122
SNU_SAM0-UNICON_gn_rlilp1fl_20190323
CMCC_CMCC-CM2-HR4_gn_rlilp1fl_20200904
CAS_FGOALS-f3-L_gn_rlilp1fl_20191007
IPSL_IPSL-CM6A-LR_gn_rlilp1fl_20180803
AS-RCEC_TaiESM1_gn_rlilp1fl_20200630